NOTE ON A PAPER OF B. GRÜNBAUM ON ACYCLIC COLORINGS

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ABSTRACT
The aim of this short note is to improve some recent results of B. Grünbaum by some remarks. We use Grünbaum's notations.

1.
Grünbaum gives an example of a planar graph with 14 vertices which is not (1,3)-colorable and mentions that this is the smallest known planar graph having this property. It is easy to verify that the graph $G_1$ in Fig. 1 below with 11 vertices is also not (1,3)-colorable. It may be shown that 11 is the minimum number of vertices (obviously one has to check only maximal planar graphs without vertices of degree 3 and there are only 20 such graphs with less than 11 vertices).

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The graph $G_2$ shown in Fig. 2 is not $(1,1,2)$-colorable, thus giving an affirmative answer to a conjecture of Grünbaum (compare remark (4) in [1]). $G_2$ contains as subgraphs a 4-clique and six copies of $G_3$ (see Fig. 3) combined in such a manner that each pair of vertices of the 4-clique is the basis pair of vertices of a copy of $G_3$. $G_3$ is a subgraph of the graph of Fig. 8 of [1] and has the property that any 4-coloring of $G_3$ yields a 4-circuit $C$ 2-colored with the colors of the basis vertices $u, v$ of $G_3$ ($C$ does not necessarily contain both vertices $u, v$ themselves). Now it is obvious that $G_2$ is not $(1,1,2)$-colorable.