A GENERAL SEARCH GAME*

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ABSTRACT

The minimax solution is found for a game in which player I chooses a real number and player II seeks it by choosing a trajectory represented by a positive function.

1. Introduction and presentation of main results

The Linear Search Game solved by Beck and Newman [1] and the extensions treated by the author [3] are special cases of the following game:

Player I chooses a number \(-\infty < t < \infty\) and player II chooses a positive function \(r(t)\) which will be called: "The search trajectory". The loss of player II is

\[
M(r(t), t) = \frac{\int_{-\infty}^{\infty} r(t + \theta)dA(\theta)}{r(t)}
\]

(1)

where \(A(\theta)\) is the distribution function of a (fixed) positive measure. We shall show that the exponential function

\[
r(t) = Ce^{bt}
\]

(2)

is a (pure) minimax search trajectory of player II. We shall also find conditions under which (2) is the unique solution, up to a multiplicative constant.

Our main results are the following:

THEOREM 1. *Let \(A(\theta), -\infty < \theta < \infty\), be any non-decreasing function. Let \(r(t), -\infty < t < \infty\), be a positive function which is integrable on every finite interval. If

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is defined for all real \( t \), then

\[
\lim_{t \to -\infty} \sup s(t) \leq \inf_{-\infty \leq b \leq \infty} \int_{-\infty}^{\infty} e^{bt} dA(\theta)
\]

and

\[
\lim_{t \to +\infty} \sup s(t) \leq \inf_{-\infty \leq b \leq \infty} \int_{-\infty}^{\infty} e^{bt} dA(\theta).
\]

**Theorem 2.** Let \( A(\theta) \) and \( r(t) \) be defined as in Theorem 1. In addition assume that \( A(\theta) \) is not concentrated at \( \theta = 0 \). Let \( g(b) \) be the Bilateral Laplace Transform of \( A(\theta) \) defined as follows:

\[
g(b) = \int_{-\infty}^{\infty} e^{bt} dA(\theta).
\]

Assume that \( g(b) \) attains its minimum at a point \( -\infty < b < \infty \) so that

\[
\int_{-\infty}^{\infty} b e^{bt} dA(\theta) = 0.
\]

If

\[
\sup_{-\infty < t < \infty} \frac{\int_{-\infty}^{\infty} r(t + \theta) dA(\theta)}{r(t)} \leq g(b)
\]

then

(a) If \( A(\theta) \) is not arithmetic*, \( r(t) = C e^{bt} \) a.s. where \( C \) is any constant.

(b) If \( A(\theta) \) is arithmetic with span \( \lambda \), \( r(t) = C(t)e^{bt} \) where \( C(t) \) is a periodic function having period \( \lambda \).

Both Theorems 1 and 2 hold for the discrete case (i.e. where \( A(\theta) \) and \( r(t) \) are replaced by positive sequences). A detailed formulation of the discrete version will be given in Chapter 4.

Chapter 5 will contain some examples which will clarify the use of the theorems. A vivid illustration of the application of the theorems for several search games is presented in [3].

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* We use arithmetic in the sense used in Feller [2], namely: A distribution \( A \) is arithmetic if it is concentrated on a set of points of the form \( 0, \pm \lambda, \pm 2\lambda, \ldots \) The largest \( \lambda \) with this property is called the span of \( A \).