SOLID STATE PHYSICS

ESTIMATING VOLUME CHANGES IN THE DIFFUSION ZONE.

2. INTERACTION OF TWO FINITE MEDIA

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We solve the problem of full diffusion homogenization of bimetallic samples of finite thickness. In the special case when the thickness of one of the materials is small, we obtain an approximate analytic solution. Numerical analysis enables us to construct the concentration distribution, the relative volume change of the sample, and also the stress and deformation distribution at various moments of time. We determine the full homogenization time and the relative volume change as a function of the relative thickness of the initial samples.

INTRODUCTION

The diffusion interaction kinetics of materials, and the consequent volume changes in the diffusion zone, depend significantly on the initial volumes or dimensions of the samples. The same applies to the nature of the stress-deformation state. In order to estimate the influence of similar factors on the mutual diffusion process, we shall consider the following problem.

PROBLEM STATEMENT

Let a planar layer of a metal A, of thickness $h_A$, be in contact with a layer of metal B, of thickness $h_B$ (Fig. 1a). Within the framework of multicomponent diffusion [1] and when the solubility of the metals in each other is unlimited, the problem is mathematically formulated as follows:

\[
\frac{\partial C_A}{\partial t} = \frac{\partial}{\partial x} \left[ D_{AA} \frac{\partial C_A}{\partial x} + D_{AB} \frac{\partial C_B}{\partial x} \right] = -\frac{\partial J_A}{\partial x} ;
\]

\[
\frac{\partial C_B}{\partial t} = \frac{\partial}{\partial x} \left[ D_{BA} \frac{\partial C_A}{\partial x} + D_{BB} \frac{\partial C_B}{\partial x} \right] = -\frac{\partial J_B}{\partial x} ;
\]

\[
r = 0; \quad J_A = J_B = 0 ;
\]

\[
x \equiv h_1 + h_2 ; \quad J_A = J_B = 0 ;
\]

\[
t = 0: \quad C_A = C_{A_1} ; \quad C_B = 0 , \quad 0 < x < h_A ;
\]

\[
C_B = C_{B_0} , \quad h_A < x < h_A + h_B .
\]

The diffusion coefficients in (1) should satisfy the inequality [2]

\[
\Delta = D_{11} D_{33} - D_{13} D_{31} > 0 .
\]
It is necessary to find the concentration distribution, and the stress and deformation distributions, in the diffusion zone as a function of time. We note that the ratio $dC_B/hC_A$ in fact characterizes the ratio of the volumes of the starting components.

**APPROXIMATE ANALYTIC SOLUTION**

In order to simplify the analytic derivation we shall consider the special case $h_B << h_A$, and we transform to a symmetric form of the problem (Fig. 1b) with the following symmetry conditions at $x = 0$:

\[
\frac{\partial C_A}{\partial t} = \frac{\partial C_B}{\partial t} = 0, \quad x = 0. \tag{5}
\]

Naturally, in the limiting case $h_B = \delta$ the solution of the problem should be similar, since the condition (2), the absence of material flux across the boundary $x = 0$, is equivalent to (5).

Let the layer of material $B$ be sufficiently thin that the spatial distribution of concentration in it may be ignored. This condition makes it possible to integrate the system of equations (1) from $\delta$ to $\delta + h$, or from $-(\delta + h)$ to $-h$. For example,

\[
\int_{-h}^{2h} \frac{dC}{\partial t} \, dx \approx \int_{-h}^{2h} \frac{\partial C_A}{\partial t} \, dx \approx J_A^{(\delta + h)} - J_A^{(\delta)},
\]

where $C_A$ is the average concentration $C_A$ in the layer $B$ of thickness $d$. Using the condition that the diffusion flux is continuous at the interface, we may transform (1) to a system of equations in the range $x \in [0, h]$, with the boundary condition at $x = h$

\[
\frac{\partial C_A}{\partial t} \Big|_{x = h} = J_A, \quad \frac{\partial C_B}{\partial t} \Big|_{x = h} = J_B \tag{6}
\]

and the symmetry condition (5). Here we have implicitly used the condition that the concentration of the components must be continuous, which is valid only in the case of unlimited solubility considered here. In all other cases, along with the material fluxes, it is only the chemical potentials of the components which must be continuous.

Further analysis of the problem is greatly facilitated by using a Laplace transform: $t \rightarrow \rho, \quad C_A \rightarrow y_1, \quad C_B \rightarrow y_2$, where $\rho$ is a complex variable, and $y$ and $z$ represent the component concentrations. After transforming to the variables $\rho, y$, and $z$, the solution involves regular differential equations with constant coefficients, which may be solved by the usual methods. In this representation space, we find

\[
y_1 = \frac{y_{10}}{\rho} + C \{ \exp(-2\kappa \rho) \exp(\kappa x \rho) + \exp(-\kappa x \rho) \} + \frac{F \{ \exp(-2\kappa \rho) \exp(\kappa x \rho) + \exp(-\kappa x \rho) \} \rho}{\rho^2},
\]

\[
y_2 = \frac{y_{20}}{\rho} - C \{ \exp(-2\kappa \rho) \exp(\kappa x \rho) + \exp(-\kappa x \rho) \} - \frac{F \{ \exp(-2\kappa \rho) \exp(\kappa x \rho) - \exp(-\kappa x \rho) \} \rho}{\rho^2}, \tag{7}
\]