Saturation of the Froissart Bound by Crossing Symmetric and Unitary Amplitudes.

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Summary. — A class of amplitudes is constructed that saturates the Froissart bound and satisfies crossing symmetry and all inelastic unitarity inequalities.

1. – Introduction.

The Froissart bound of the elastic-scattering amplitude (1) has been derived on the basis of axiomatic quantum field theory (2) and there is no question that any specific field theory of hadrons has to obey this bound. The proof can be given on the basis of rather few properties of the amplitude (2) and it might be possible that this bound can be improved if more constraints (which are satisfied by quantum field theory) are taken into account. This problem is not only of academic interest, since recent experimental results on pp, p̅p and πp scattering are compatible with the saturation of this bound (3).

fact, one can expect more experimental information about this question pretty soon from p\bar{p} collisions at the CERN-SPS.

The aim of this paper is to show that the Froissart bound cannot be improved on the basis of the general constraints analyticity, crossing symmetry and inelastic unitarity alone. The proof is done by the construction of ππ scattering amplitudes which saturate the Froissart bound. The ππ scattering process is chosen for two reasons:

1) it has maximal constraints due to crossing symmetry, and
2) the particles have no spin.

Concerning the experiments, it is desirable to construct amplitudes for spin-\frac{1}{2} particles. This is possible with essentially the same methods as presented here and will be done in another publication (4).

The scattering amplitudes \(A(s, t)\) is written with the usual Mandelstam variables \(s, t\) and \(u\), the mass of the particles is renormalized to unity, and, therefore, \(s + t + u = 4\). The amplitude is holomorphic and polynomially bounded

\[
|A(s, t)| < \text{const} \ (1 + |s| + |t|) \quad \text{in the domain}
\]

\[
\mathcal{C}_{\text{cut}} = \{(s, t) \in \mathbb{C}^2 | s \notin [4, \infty), t \notin [4, \infty), s + t \notin (-\infty, 0)\}.
\]

Crossing symmetry is the total symmetry in the variables \(s, t\) and \(u = 4 - s - t\),

\[
A(s, t) = A(t, s) = A(s, u) = A(u, s) = A(t, u) = A(u, t).
\]

(The discussion of isospin crossing symmetry is postponed to sect. 4.)

The partial-wave decomposition on the physical cut \(s > 4\) is written as

\[
A(s + i0, t) = \sqrt{\frac{s}{s-4}} \sum_{l=0}^{\infty} (2l + 1) a_l(s) P_l \left(1 + \frac{2t}{s-4}\right)
\]

with

\[
a_l(s) = \left[s(s-4)\right]^{-l} \int_{4-s}^{0} A(s + i0, t) P_l \left(1 + \frac{2t}{s-4}\right) dt.
\]