INVESTIGATION OF THE STRUCTURE OF THE SECONDARY FLOW IN BLADE SYSTEMS OF FRANCIS TURBINES

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At present the level of the maximum efficiency of Francis turbines has reached 95-97% and their further improvement requires the use of more accurate calculation methods. In connection with this, three-dimensional models of fluid flow in blade systems have come to be used more widely. In particular, quasi-three-dimensional methods, which reduce the calculation of a three-dimensional flow in a hydraulic machine to the combined solution of mutually related two-dimensional problems — the axisymmetric problem of calculating the flow averaged in the circumferential direction in the water passage of hydraulic machines and the problem of calculating flow past cascades of airfoils in a layer of variable thickness on stream surfaces of an average flow $S_{1av}$ — made a good showing in engineering practice.

The flow obtained as a result of solving the indicated two problems is called the main flow. To obtain a rigorous solution of the three-dimensional problem it is necessary to solve a third problem of determining the additional (secondary) flow normal to the surfaces $S_{1av}$. Such a division of the actual three-dimensional flow into imaginary ones makes it possible in many cases to find the structure of the three-dimensional flow in a hydraulic turbine by an order more simply (from the viewpoint of realization and labor intensity) and, what really matters, sufficiently accurately, and also makes it possible to estimate the hydrodynamic indices of the turbine on the basis of quasi-three-dimensional methods that have been approved and, as already mentioned, have made a good showing.

Many works (e.g., [1--4]) are devoted to theoretical investigations of secondary flows.

The vortex model of movement of an ideal fluid, the foundations of which were laid long ago by N. E. Zhukovskii (Joukowski), gained popularity in the theory of hydraulic machines. We note work [2], which gives the formulation and numerical realization on a computer of the third two-dimensional problem of calculating the secondary flow of an ideal fluid on axisymmetric surfaces $S_3$ normal to a family of stream surfaces $S_{1av}$. In [5] a simple method was proposed for realizing the indicated problem, which makes it possible to find sufficiently accurately the distribution of the parameters of the secondary flow on the surface of a runner blade without determining the velocity field in its interblade channel. This reduces the labor intensity of the calculation by an order. The method presented in [5] was subsequently improved [6]. We will point out the main theoretical and physical aspects of its formulation.

Let us consider that the direct quasi-3D problem is solved and the parameters of the main flow in the water passage of the turbine are known. We will examine a natural coordinate system $q_1, q_2, q_3$ (Fig. 1) in which the coordinates $q$ are directed in the meridional plane along the generatrices of the axisymmetric stream surfaces $S_{1av}$, coordinates $q_2$ are normal in this plane to $q_1$, and coordinate $q_3$ coincides with the circumferential direction of rotation of the runner. We will represent the relative velocity $\overrightarrow{W}$ at any point of the blade surface as the sum of the main velocity $W_{0}$ located on surface $S_{1av}$ and the additional velocity $W_{3} (C_{1g}, C_{2g}, C_{3g})$. Taking into account flow past the blade surface $\overrightarrow{W} \overrightarrow{n} = 0$, we can write that

$$W_{30} = C_{10} \cot g \beta ; \quad C_{3g} = C_{1g} \cot g \beta + C_{2} \cot g \alpha ;$$

were $\cot g \beta = -n_1/n_3$ and $\cot g \alpha = -n_2/n_3$ are geometric characteristics of the blade surface; $n_1$, $n_2$, $n_3$ are projections of the normal vector $n$ toward it onto the coordinate axes $q_1, q_2, q_3$. Analogously, the energy of the fluid in relative motion $E$, the pressure $P$, and the components of the vorticity vector $\overrightarrow{\Omega}$ ($\Omega_1$, $\Omega_2$, $\Omega_3$) = $\text{rot} \overrightarrow{C}$ are represented as the sum of the corresponding quantities of the main and additional flows, in which connection the energy $E_0$ of the main flow keeps a constant value on the surfaces $S_{1av}$ and the projections $\Omega_{20} = \text{rot}_2 \overrightarrow{C}_0$ of the main flow are equal to zero.
To solve the problem we use the equations of motion and continuity in the form

\[ \vec{W} \text{ rot } \vec{C} = \text{grad} \, E ; \]  
\[ \text{div } \vec{W} = 0 , \]  

where \( \vec{C} = \vec{W} + \vec{U} \) are vectors of the absolute, relative, and transport velocities; \( U = \omega r, \) \( \omega \) is the rotational speed of the runner;

\[ E = (W^2 - U^2)/2 + \frac{P}{\rho} + gz. \]

Equation (2) can be transformed for points located on the blade surface to the form

\[ \frac{\partial E}{H_1 \partial q_1} = -C_2 (\vec{\Omega} \cdot \vec{n})/n_3 ; \quad \frac{\partial E}{H_2 \partial q_2} = C_1 (\vec{\Omega} \cdot \vec{n})/n_3, \]  

where

\[ \frac{\partial E}{H_1 \partial q_1} = \frac{\partial E}{H_1 \partial q_1} + \tan \beta \frac{\partial E}{r \partial q_3} ; \]
\[ \frac{\partial E}{H_2 \partial q_2} = \frac{\partial E}{H_2 \partial q_2} + \tan \alpha \frac{\partial E}{r \partial q_3} \]

are derivatives of the energy \( E \) along the blade surface.

In addition to the indicated equations, to determine the secondary flow we use also equations of motion of the fluid (2), (3) averaged in the circumferential direction and the expression for the average vorticity [7]:

\[ \vec{\Omega} = \frac{1}{3} \text{rot} (\vec{\Omega}) + \frac{1}{r \partial} \Delta \left[ \frac{1}{n_3} \vec{\nabla} \vec{C} \right] \].  

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