PROCEDURE FOR NUMERICAL INVESTIGATION OF FORCED VIBRATIONS
OF THREE-DIMENSIONAL STRUCTURES WITH NONLINEAR ELASTIC ELEMENTS
BASED ON THE FINITE-ELEMENT METHOD

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We describe a numerical procedure which allows one to investigate forced vibrations of complex three-dimensional structures with nonlinear elastic elements, develop a computational dynamic model on the basis of the finite-element method and the method of generalized coordinates, and investigate vibrations of an actual structure of a load-bearing mounting of an aircraft engine. We also analyze the effect of nonlinear elastic elements on the dynamic behavior of the structure and evaluate the efficiency of their application.

Special-purpose nonlinear elastic elements whose introduction in structures change their amplitude-frequency characteristics and prevent resonance phenomena are extensively used in numerous branches of industry and in machine building [1,2]. To give an adequate description of the behavior of such structures, it is necessary to use combined dynamic models including linear and physically nonlinear elements.

We propose a numerical procedure which makes it possible to take into account the work of nonlinear elastic elements in the analysis of forced vibrations of 3D structures.

In the process of discretization, 3D structures are represented as sets of finite elements.

Nonlinear elastic elements are simulated by rod-type finite elements (Fig. 1) operating under the conditions of tension and compression. According to the stress-strain diagram for nonlinear elastic elements (Fig. 2), the variation of the potential strain energy of these elements is represented in the form of the sum of its elastic and nonlinear components:

$$\delta \tilde{\Pi} = (\tilde{K}(u)\delta u = (\tilde{K}_L u, \delta u) + (\tilde{K}_N(u)u, \delta u),$$

where $\tilde{K}$ is the stiffness matrix of nonlinear elastic elements, $\tilde{K}_L$ is the stiffness matrix of nonlinear elastic elements simulated by elastic elements, and $\tilde{K}_N$ is the stiffness matrix appearing as a result of nonlinearity.

In constructing a discrete dynamic model, we form a linear stiffness matrix of finite elements of the 3D structure, namely,

$$K = K_L + \tilde{K}_L,$$

where $K$ is the stiffness matrix related to all finite elements and $\tilde{K}$ is the stiffness matrix related to all finite elements except nonlinear elastic elements.

Thus, the dynamic model for the description of forced vibrations of 3D structures which includes nonlinear elastic elements in its finite-element formulation takes the form

$$M \frac{d^2 u}{dt^2} + C \frac{du}{dt} + Ku + \tilde{K}_N(u)u = R(t),$$

where $u(t) = (u_1(t), u_2(t), ..., u_n(t))^T$ is the vector function of nodal displacements of dimension $n$, $n$ is the number of degrees of freedom of the system, $M$ and $C$ are, respectively, the matrices of inertia and damping of dimension $n \times n$, and $R(t) = (R_1(t), R_2(t), ..., R_n(t))^T$ is the vector function of external nodal forces.
Fig. 1. Location of a nonlinear elastic element prior to (I) and after (II) deformation.

Fig. 2. Deformation of a nonlinear elastic element (solid and dashed lines correspond to experimental data and the data obtained by its simulation with a finite elastic elements, respectively).

Suppose that, prior to deformation, a nonlinear elastic finite element occupies position I and its axis coincides with the segment AB (Fig. 1). We denote the coordinates of the nodes A and B in the global coordinate system by \( x_{bi} \) and \( x_{di} \), respectively. In the deformed state II, the coordinates of the nodes A and B are \( x_{bi} + u_{bi} \) and \( x_{di} + u_{di} \), respectively, (\( u_{bi} \) and \( u_{di} \) are the projections of the displacement vectors \( \overline{u}_b \) and \( \overline{u}_d \) in the global coordinate system \( x^i \), \( x^j \), and \( x^k \)). Moreover, in the system of principal directions, \( a^{i'} \), \( a^{j'} \), and \( a^{k'} \), the projections of the vectors \( \overline{u}_b \) and \( \overline{u}_d \) are \( u_{bj} \) and \( u_{dj} \) (\( j' = 1, 2, 3 \)). The components of the projections of displacement vectors in the systems of global coordinates and principal directions are related by the following formulas:

\[
\begin{align*}
    u_{bi} &= \sum_{j=1}^{3} L_{bi',j} u_{bi'} \\
    u_{di} &= \sum_{j=1}^{3} L_{di',j} u_{di'}
\end{align*}
\]  

(4)

where \( L_{bi',j} \) and \( L_{di',j} \) are the matrices of direction cosines. The length of the element prior to deformation has the form

\[
    l^e = \left[ \sum_{i=1}^{3} \left( x_{di'}^e - x_{bi'}^e \right)^2 \right]^{1/2},
\]

(5)

whereas after deformation, it is determined as

\[
    \overline{l}^e = \left[ \sum_{i=1}^{3} \left( x_{di'}^e - x_{bi'}^e \right)^2 \right]^{1/2}.
\]

(6)

The elongation of the nonlinear elastic finite element in terms of displacements of the global coordinate system is given by the expression

\[
    \Delta l^e = \overline{l}^e - l^e = l^e \left[ \left( 1 + \sum_{i=1}^{3} \left| \frac{\Delta u_{i'}}{l^e} \right| \right)^2 + 2 \sum_{i=1}^{3} \frac{\Delta u_{i'}}{l^e} \cos \alpha_i^e \right]^{1/2} - 1,
\]

(7)

where \( \Delta u_{i'} = u_{di'} - u_{bi'} \), and \( \cos \alpha_i^e = (x_{di'}^e - x_{bi'}^e)/l^e \).