Evolution of Nonperiodic Forms in Geological Folds

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An elastic layer resting on a visco-elastic (Maxwell) foundation is used to model the initiation and subsequent development of a geological fold. This study focuses on the effect of nonlinear terms, of both geometric and material origin, in the formulation of the governing differential equation. The result, during the initial elastic phase of deformation, is to admit the infinite variety of shapes familiar to the field of localized buckling. Their continued effect during the evolutionary phase allows the development of nonperiodic forms hitherto ignored by linear stability analyses. Numerical results are presented for two visco-elastic models subjected to loading conditions of constant end displacement and constant rate of end displacement.

KEY WORDS: localization, buckling, nonlinearity, numerical solutions.

INTRODUCTION

A simple beam model is used in this paper to study the growth of geological folds and to highlight the existence of localized buckled shapes which are comparatively new in the context of geological folding. Much of the previous work in this field has been inspired by the work of Biot (1965) and is based on critical loads obtained from a linear instability analysis. Such analyses typically consider linear rheological properties and geometries, and are strictly valid only for infinitesimal deflections. By incorporating nonlinear behavior in the formulation of the system and by considering nonperiod buckle patterns, the analysis presented here goes beyond the initial phase of buckling and accounts more readily for the diversity of geometrical forms observed in nature.

Many previous instability analyses have considered the buckling of layers under conditions of constant load (Biot, 1965; Kerr, 1969). This is equivalent to gravity (or dead) loading of vertical column structures and may lack meaning...
in the context of geodynamics. A more realistic loading condition, relevant to plate tectonics, results from the impact of a mass moving with constant velocity, on an initially horizontal layer. By considering the mass to be infinite, a step velocity of the layer end is prescribed with axial displacement increasing linearly from the moment of impact. This is akin to rigid loading in the context of structural engineering. Under these conditions total collapse of the structure is inevitable, and interest centers on the mode of collapse and the ensuing change of load with time.

This paper is not the first to explore the influence of nonlinearities in this time-dependent buckling problem. Mühlhaus, Hobbs, and Ord (1994) and Hunt, Mühlhaus, and Whiting (1996) have examined the effect of constant axial displacement on the buckling of an elastic layer. The first authors considered the evolution of periodic forms in an elastic plate, of length L, embedded in a purely viscous (Newtonian) medium. They determined that the initial stages of fold evolution were governed by Biot's (1965) dominant wavelength, determined from a linear stability analysis, and that the fold then proceeded through various transitional stages as $t \neq 0$, eventually evolving to a single half-sine wave over the full length of the plate. The latter authors drew attention to the possibility for nonperiodic forms and localization within the geological framework. They focused on the deformation of an elastic layer supported by a visco-elastic (Maxwell) wavelength-dependent foundation. They concluded similarly that, as the load dropped asymptotically to zero, the fold pattern would progress through a series of localized states, ultimately adopting the same half-sine wave.

The significance of localization may be appreciated by considering the nature of the end-shortening, $\Delta$, for an infinitely long inextensible layer. For a periodic buckle pattern of finite amplitude, $\Delta$ is likewise infinite, but for a localized buckle it is finite. The energy required for this deformation process, therefore, is finite. For long structures under constant or slowly differing end displacement, the deformation may be envisaged as a succession of localized forms in the complete absence of periodicity.

The structure of this paper is as follows. First, the assumptions used to develop the simple beam models are described and the linear forms of the governing differential equations are derived. Analysis of the linearized equations then is presented, which is important in characterizing the solution behavior when deflections are small. Next, the numerical procedure used to solve the differential equations is outlined. Finally, evolving buckle patterns for two visco-elastic models are presented. The first (Model A) includes geometrically nonlinear terms in the formulation of the system, and is capable of modeling large deflections (Thompson and Hunt, 1973). The second (Model B) uses linear beam theory coupled with a nonlinear foundation, and is valid for small, but finite, deflections.