Scattering Problems with Spin-Orbit Couplings.

T. Regge and M. Verde

Istituto di Fisica dell'Università - Torino
Istituto Nazionale di Fisica Nucleare - Sezione di Torino

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Summary. — With the aim of treating the scattering of neutrons by deuterons in presence of tensor forces or spin-orbit couplings we give here a variational principle which does not use Green's tensor. The method is an extension of that due to Hultén for the case of two bodies and central forces. In section 1 we discuss the two-body case with tensor potentials, in section 2 the three-body case with central forces and finally we extend our treatment in the last section to the more general case of neutron-deuterons scattering with spin-orbit coupling.

Introduction.

The purpose of this investigation is to give a contribution to some collision problems of nucleons or complex nuclei. We are chiefly interested in the case of forces for which the total intrinsic spin is not a good quantum number.

We begin in section 1 with a variational method which allows the evaluation of the phases for the scattering of two nucleons in presence of the tensor force. This method is an extension of that given by Hultén (1) for central forces in his second formulation. In the same section we discuss the connection of the new method with the old one due to Schwinger (2).

For collisions between complex nuclei we consider the specific case of neutron-deuteron scattering. In section 2 we have limited ourselves to central

forces only, in order to formulate the variational method similar to that given by one of us (3), but in a form by far more advantageous for the actual calculation of the phases. This is again an extension of Hulthen's method for two bodies mentioned before.

Collision between complex nuclei can be treated in the same manner with only slight formal complications.

In the third section we give a further generalisation of the results obtained eliminating the restriction of central forces. As in section 2 we consider the neutron-deuteron case, but our considerations are of more general validity. We emphasize also the circumstance that possible many-body potentials may be fitted into the same formulation without bringing on new difficulties.

1. — The two-body case.

It will be sufficient to consider the triplet scattering only. The tensor force is indeed active merely for \( S = 1 \) and mixes for a given total angular momentum \( J \) the two orbital momenta \( l = j \pm 1/2 \).

We call, as usually, with \( u \) and \( w \) the radial components of the wave function corresponding to the two angular momenta \( l = j \pm 1/2 \). The equation of motion then gives the well known system of coupled differential equations for \( u \) and \( w \):

\[
\begin{pmatrix}
L_{j-1} & 0 \\
0 & L_{j+1}
\end{pmatrix}
\begin{pmatrix}
u \\
w
\end{pmatrix}
= \begin{pmatrix} f & g \\ g & h \end{pmatrix}
\begin{pmatrix}
u \\
w
\end{pmatrix},
\]

where \( L_j \) is the differential operator,

\[
L_j = \frac{d^2}{dr^2} + k^2 - \frac{j(j+1)}{r^2},
\]

is the kinetic energy for the relative motion, \( f, g, h \), are functions of \( r \). \( g \) is proportional to the tensor potential only, whereas \( f \) and \( h \) include also the central potential. The system (1) reduces to a set of two independent ordinary differential equations in absence of tensor force \((g = 0)\).

The radial component \( v \) belonging to the orbital momentum \( l = J \) evolves separately because it corresponds to a different parity than \( u \) and \( w \).

\( v \) obeys an ordinary differential equation for which it is possible to formulate the same variational principle as in the case of central forces.

We have therefore to consider merely the system (1).