Bound and Rebound States.

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Summary. — In relativistic quantum theory, bound states generate forces in the crossed channel; such forces can affect the binding and self-consistent solutions should be sought for the bound-state problem. We investigate how self-consistency can be achieved by successive approximations, in a simple scalar model and with successive relativistic eikonal approximations (EAs). Within the generalized ladder approximation, some exact properties of the resulting «first generation» bound states are discussed. The binding energies in this approximation are rather small even for rather large values of the primary coupling constant. The coupling of the constituent particles to the first-generation reggeon is determined by a suitable EA and a new generalized ladder amplitude is constructed with rungs given either by the primary gluons or by the first-generation reggeons. The resulting new (second-generation) bound states are found in a reggeized EA. The size of the corrections to the binding energies due to the rebounding effects is surprisingly large. The procedure is then iterated, so as to find—again in an EA—the third-generation bound states. The procedure is found to be self-consistent already at this stage; the third-generation bound states coincide with those of second generation, and no further rebounding takes place in the higher iterations of the approximation method. Features—good and bad—of the model are discussed, as well as the possible relevance of rebounding mechanisms in hadron dynamics.

1. — Introduction.

The problem of bound states represents one of the least understood aspects of relativistic quantum physics. In particular, a sensible field-theoretic approach to deep-binding phenomena is conspicuously in want. In the present
paper, we speculate on what might be expected from a field theory of composite particles: we propose a dynamic mechanism for deep binding and construct a simple field-theoretic model, where the mechanism is investigated in some detail.

To summarize our main concern, consider a local "primary" coupling $g_0\bar{Q}QB$ between constituent particles ("quarks" $Q$ and "antiquarks" $\bar{Q}$) and "gluons" $B$, and assume that, in a ladder approximation, this coupling generates $\bar{Q}Q$ bound states $\pi_0$. Now, these bound states "of first generation" will induce a secondary effective interaction, $g_0\bar{Q}Q\pi_0$, which can combine with the primary (ladder-approximated) coupling, so as to yield new $\bar{Q}Q$ bound states "of second generation", $\pi_1$, whose binding forces are due to either primary gluons or first-generation bound states. The second-generation bound states can, in turn, further correct the binding forces through a new secondary induced interaction, $g_0\bar{Q}Q\pi_1$, and so on: such a rebinding mechanism could operate any number of times, and the final answer for what are the true bound states of the theory might correspond to a limit, as $n \to \infty$, of the $n$-th generation bound states, $\pi_{n-1}$. While the rebinding effects might be negligible for a weak primary interaction, they might sizably affect the result for strong primary coupling or, more generally, in case of deep binding.

In any case, it is clear that the true bound states of the theory, $\pi$, must correspond to self-consistent solutions of the bound-state problem, whereby the crossed-channel $\bar{Q}Q$ interaction induced by $\pi$-exchange is already taken into account when solving the problem of $\bar{Q}Q$ binding to form the $\pi$ themselves. This self-consistency could be achieved in a number of ways, e.g. by a "pure bootstrap", where the primary interaction is altogether absent, or by an "infinite regression" of the type described above, or even after a finite regression, whereby the bound states of some finite generation are already self-consistent, and no further rebinding occurs.

We will be concerned with the possible size of the rebinding effects, as well as with other qualitative features of the secondary interactions, particularly with the mechanism leading to self-consistency.

In nonrelativistic potential theory, secondary rebinding is absent because of the absence of crossing symmetry. Since the effects are due to the forces generated by the bound states in the crossed channel, rebinding is also neglected in the simplest generalizations of potential theory, such as ladder-type approximations for the primary coupling. One might hope that the rebinding forces might turn out to be negligible in cases of practical interest, so that the first-generation bound states might, after all, prove reasonable guides for understanding deep-binding phenomena. However, we will argue below that deep binding does not occur in the primary generalized ladder approximation, and that in this approximation the binding is weak even for rather large values of the primary coupling constant $g_0$. Therefore, it appears that a realistic local Lagrangian theory of deep binding should come to grips with the intri-