Unifying the $SU_5$ Grand Unification Theory.

P. JACOB, H. SALLER and H. SCHNEIDER

Max-Planck-Institut für Physik und Astrophysik - München, B.R.D.

(rievuto l'11 Novembre 1980)

Summary. — One left-handed fermion field $\chi$ in the fundamental decuplet representation of $SU_5$ is proposed as the common building block of the phenomenological fields of the $SU_5$ grand unification theory. In a first step, bilinears of the $\chi$ field can form the gauge and the Higgs bosons. In a second step, the quarks and leptons can arise as bound states of the composite Higgs field and the basic $\chi$ field. The $SU_5$ symmetry will be broken spontaneously by a condensate of the Higgs and of the gauge bosons. If the dimension $\frac{5}{2}$ is assigned to the basic field $\chi$, the correct canonical dimensions for the phenomenological fields will arise.

1. — Introduction.

The proliferation of phenomenological fields (quarks and leptons in different families, gauge and Higgs fields) has stimulated attempts for grand unification approaches which go into different directions.

« Additive » unification schemes (1) $(SU_3, SO_{10}, E_6, \text{etc.})$ treat all the fields as fundamental degrees of freedom and, therefore, utilize large gauge groups containing the relevant subgroups $SU_{3,c}, SU_{2,l}$ and $U_{1,t}$. Via Higgs potentials mass scales are introduced which allow the distinction of different regions where different subsymmetry schemes are relevant. « Radical » unification schemes (2,3) try to explain the groups $SU_{3,c}, SU_{2,l}$, and $U_{1,t}$ as diff-

(1) See, e.g., D. V. NANOPULOS: preprint TH 2896-CERN (1980), and quoted references therein.
ferent manifestations of an underlying simpler structure. In between there
are subquarks models (4) that replace part of the phenomenological symmetries
by new symmetries on the basic level.

In this paper we propose a substructure in which a local $SU_5$ group is as-
sumed to be the relevant symmetry. In this case the $SU_5$ gauge fields $A_\mu^a$ seem
to be the natural starting point. The existence of spin-$\frac{1}{2}$ fermions on the
phenomenological level may be taken as a hint that the gauge fields have an
underlying bilinear fermionic structure. Therefore, a $SU_5$ fermion field $\chi$
(the «square root» of the vector gauge field (5)) is a more natural starting
point to implement a $SU_5$ symmetry which manifests itself in vector gauge
fields, fermions and Higgs fields. A closer analysis shows that a fermion field $\chi$
in the fundamental decuplet representation with subcanonical dimension $\frac{1}{2}$
fulfils both the group-theoretical and the (naive) dimensional conditions to
build the phenomenological fields (with the exception of the Higgs field in the
adjoint representation, which can possibly be replaced by a gluon con-
densate).

Whereas we have reduced the number of representations of the unification
group to one fundamental representation, there is no simplification of the group.
The whole $SU_5$ is taken to be basic and no part is structural in the sense that
it arises only on a composite level. At that stage the approach poses the
question in a clearer form why $SU_5$ should be relevant as a basic group. In our
construction we find the possibility for some canonical fields that seem to
have no counterpart in Nature and that would introduce anomalies in the
effective Lagrangian, if they exist. This may be a hint for a simpler group
than $SU_5$ at the fundamental level. On the other hand, for the self-interac-
tion part of the Lagrangian of the subcanonical field to be nonzero, at least
two internal degrees of freedom of the basic fermion field have to be present.

This paper shows—more in a programmatic form—the possibilities of such
an approach and leaves many open questions mostly connected with the dif-
ficulties to master the bound-state problem. Nevertheless, there is no complete
waste land in this respect. In connection with the subcanonical theories there
are some attempts to understand the gauge field bound-state problem (6)
(especially its mass-zero aspect), to consider Bethe-Salpeter-like equations for
scalar bound states (6) (Higgs fields), and to investigate the indefinite-metric
problems (7) introduced by the subcanonical fields.

---

(5) H. P. DÜRR and N. U. WINTER: Nuovo Cimento A, 70, 467 (1970); H. SALLER:
(6) W. HEISENBERG: Introduction to the Unified Field Theory of Elementary Particles
(7) W. HEISENBERG: Nucl. Phys., 4, 532 (1957); C. C. CHIANG and H. P. DÜRR: