Re-Examination of the $\pi NNN-NNN$ Problem.

G. CATTAPAN (1), L. CANTON (1) and J. P. SVENNE (2)

(1) Dipartimento di Fisica ''Galileo Galilei'' dell'Università - Padova
INFN, Sezione di Padova - via F. Marzolo, 35131 Padova
(2) Department of Physics, University of Manitoba
Winnipeg Institute of Theoretical Physics - Winnipeg, Manitoba, Canada R3T 2N2

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Summary. — We discuss the dynamical equations for the $\pi NNN-NNN$ system. We introduce the Afnan-Blankleider-Avishai-Mizutani equations in a very direct and immediate way, using the diagrammatic method and special properties of the four-body transition operators. The connection between this four-body unitary theory and a more phenomenological model for the $^3\text{He} + \pi \rightarrow NNN$ process, recently developed by the authors, is established.

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1. - Introduction.

Pion absorption and scattering reactions on very light ($A \leq 3$) nuclei have recently attracted considerable interest from both the experimental and the theoretical point of view. On the experimental side, high-precision exclusive studies have been performed, to test the reaction mechanisms involved and to extract information upon the properties of the underlying strong interaction[1, 2]. The situation is far less satisfactory on the theoretical side, however. In particular, whereas sophisticated three-body models have been formulated and used to perform extensive numerical calculations for the $\pi NN$ system[1], to date only highly phenomenological approaches have been applied to scattering and absorption on $A = 3$ nuclei.

Absorption studies have been mainly restricted to the two-body quasi-free absorption model, in which the third nucleon acts as a spectator. There are two levels of approximation implied by this model. On a first level, the $\pi NN$ interactions in the initial state and/or the three-nucleon final-state correlations are completely disregarded[3, 4], or at most taken into account with respect to the absorbing pair only[5-7]. On a more fundamental level, there is a need to couple the absorption channel to all the available channels ($\pi$ production, scattering, etc.) so that all these processes can be described by a unique, unitary theory. This need to describe the dynamics with explicit allowance of pion absorption and production cannot be met by the quasi-free two-body approximation. This state of affairs is extremely
unfortunate, since \( A = 3 \) nuclei represent a much richer system than the \( \pi NNN \) one, because of the presence of two possible target bound states and a much wider spectrum of initial and final channels. In particular, items such as absorption on both isoscalar and isovector nucleon pairs or the coherent absorption on three nucleons can be studied, which are obviously beyond the possibilities offered by the \( \pi NNN \) system.

Recently, we have introduced a model for pion absorption on \( A = 3 \) nuclei [8, 9], in which the correlations among the three nucleons both in the initial and in the final channel can be described exactly by means of three-body Faddeev theory. At the same time, four-body kinematics are handled in the proper way, at least in the non-relativistic limit for the participating nucleons. Pion absorption is described through an effective operator, which can accommodate both two-nucleon mechanisms (as described, e.g., through the rescattering model [10]) and possible three-nucleon processes. When the correlated pairs of nucleons are described by means of Weinberg quasi-particle states, the basic reaction mechanisms can be identified, and classified according to the number of participating nucleons [8]. At the same time, owing to the employment of modern few-body techniques, the prescriptions due to the identity of the three nucleons can be exactly taken into account [9].

The main approximation of our model consists in the neglecting of the interaction between the incoming pion and the target, as well as of the explicit coupling with pion absorption-emission processes in the final state. To overcome these limitations, a consistent unitary four-body theory of the \( \pi NNN \) system would be required. In other words, one would have to generalize the unitary coupled \( \pi NNN - \pi NN \) models [1] to the \( \pi NNN - \pi NN \) case, much in the same way as the three-body Faddeev theory has been extended to the four-body case. This turned out to be a formidable task. To our knowledge, the only successful attempt up to now has been performed by Avishai and Mizutani (AM) [11, 12] who generalized the elegant approach of Grassberger and Sandhas (GS) [13, 14] to the coupled \( \pi NNN - \pi NN \) system. Starting from disconnected equations for \( \text{three-cluster} \leftrightarrow \text{three-cluster} \) transitions, AM applied the GS quasi-particle method to get coupled-channel equations of Lippmann-Schwinger type. As a second step, they apply the original GS trick to get connected-kernel equations, namely they distribute the potential matrix operator appearing in the LS equations over two-cluster partitions. Differently from the standard four-body case, however, in the \( \pi NNN \) problem this trick fails in producing fully-connected-kernel equations, because of the coupling of the four-body \( \pi NNN \) space to the \( NNN \) one (a situation obviously due to pion creation-absorption processes). As a consequence, AM have to resort to a two-potential procedure to get well-behaved scattering equations. Their final result, although in principle exact, is nonetheless extremely involved. Here, we do not intend to enter the extremely difficult question of how to guarantee four-body connectivity in the presence of absorption under the simplest conditions. Rather, we show how our non-unitary approach can be derived from an exact theory for the coupled \( \pi NNN - \pi NN \) system, since in this derivation we can focus on the contributions still not included in our model but certainly contained in the rigorous dynamical theory. This can shed more light on the nature of the approximations implied by the model.

We shall show that the basic AM three-cluster equations can be recovered through a consistent generalization of the graphical method originally developed by Afnan and Blankleider for the \( \pi NN - \pi NN \) case [15]. In particular, the renormalization of the nucleon lines and the dressing of the \( \pi NN \) form factors will be tackled before the