GENERALIZED INVERSE GROUP OF SIGNAL AND ITS
IMPLEMENTATION WITH NEURAL NETWORKS*

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Abstract A new concept, the generalized inverse group (GIG) of signal, is firstly proposed and its properties, leaking coefficients and implementation with neural networks are presented. Theoretical analysis and computational simulation have shown that (1) there is a group of finite length of generalized inverse signals for any given finite signal, which forms the GIG; (2) each inverse group has different leaking coefficients, thus different abnormal states; (3) each GIG can be implemented by a grouped and improved single-layer perceptron which appears with fast convergence. When used in deconvolution, the proposed GIG can form a new parallel finite length of filtering deconvolution method. On off-line processing, the computational time is reduced to \(O(N)\) from \(O(N^2)\). And the less the leaking coefficient is, the more reliable the deconvolution will be.

Key words Signal processing, Neural networks, Generalized inverse group, Deconvolution

I. Introduction

Theories on inverse of signals are very useful to deconvolution, signal reconstruction, inverse filtering etc., which has been one of commonly interesting research projects for decades to many researchers in disciplines at engineering and mathematical and physical science fields[1,2].

For near 50 years, the time domain deconvolution has been mainly developed based on the single delta inverse theory of signals[1,3,4]. The single delta inverse deconvolution and the least mean square deconvolution have found many good use. As the advances of intelligent information processing, especially for more and more increasing systolic array processing, neural network based processing and parallel distributed processing, more attentions are paid to intelligent, reliable, parallel and real-time processing rather than reduction of computational complexity only. Such a consideration brought insight into the multi-delta inverses[5-6] and the GIG of signal.

In this paper, a new concept, GIG of signal, is firstly proposed and its properties and application are presented. Leaking coefficients are worked out for measuring the performance of GIG, which is different from either the single delta inverse or the multi-delta inverse. A special neural network is designed for implementation of GIG, which is also simplified by

*Supported partly by Natural Science Foundation of China and Aviation Science Grant of China.
II. Inverses and Inverse Group of Signals

Consider signal \( x(n)(-\infty < n < \infty) \), we at first give following definitions.

**Definition 1** Signal \( g(n) \) is called as the inverse of a given signal \( x(n) \), if

\[
x(n) \ast g(n) = \delta(n)
\]

where \( \delta(n) \) is a unit delta function.

**Definition 2** Signal \( g^{(k)}(n)(-\infty < n < \infty) \) is defined as the \( k \)-step delayed inverse of \( x(n) \), when

\[
x(n) \ast g^{(k)}(n) = \delta(n - k), \quad k = (0, 1, \cdots)
\]

**Definition 3** The signal set of the \( k \)-step delayed inverse of \( x(n) \),

\[
\{ g^{(k)}(n), \quad k = 0, 1, 2, \cdots \}
\]

is called as the inverse group of signal \( x(n) \).

Now, we give following theorems.

**Theorem 1** If the Z transformations of both signal \( x(n) \) and \( g^{(k)}(n) \) exist and are denoted by \( X(Z) \) and \( G^{(k)}(Z) \) respectively, then

\[
G^{(k)}(Z) = Z^{-k} / X(Z)
\]

**Proof** Because \( g^{(k)}(n) \) is the \( k \)-step delayed inverse signal of \( x(n) \), then

\[
g^{(k)} \ast x(n) = \delta(n - k)
\]

Thus, the Z transformations of both sides of the above equation then become

Right Side: \( \sum_{n=-\infty}^{\infty} \delta(n - k)Z^{-n} = Z^{-k} \)

Left Side: (from the convolution theorem):

\[
\sum_{n=-\infty}^{\infty} \left[ g^{(k)}(n) \ast x(n) \right] Z^{-n} = \left[ \sum_{n=-\infty}^{\infty} g^{(k)}(n)Z^{-n} \right] \cdot \left[ \sum_{n=-\infty}^{\infty} x(n)Z^{-n} \right]
\]

\[
= G^{(k)}(Z) \cdot X(Z)
\]

Therefore, Theorem 1 is proved.

**Theorem 2** If \( x(n) \) is a \( N \)-point \( (N \geq 2) \) finite length of signal, then \( g^{(k)}(n) \) is infinite length of signal and vice versa.

**Proof** Assume \( x(n) \) \( (n = 0, 1, 2, \cdots, N - 1) \) is a \( N \)-point finite length of signal, then

\[
X(Z) = \sum_{n=0}^{N-1} x(n)Z^{-n}
\]

is a \( N - 1 \) order polynomial of \( Z \). Assume the \( N - 1 \) zeros of \( X(Z) \) are \( z_1, z_2, \cdots, z_{N-1} \), then \( X(Z) \) maybe rewritten as

\[
X(Z) = A \prod_{i=1}^{N-1} (1 - z_iZ^{-1})
\]