DETECTION IN WEIBULL AND 
LOG-NORMAL NOISES

Zhu Zhaoda (朱兆达)
(Nanjing Aeronautical Institute, Nanjing, China)

Abstract

The discrete-time detection of narrowband coherent and incoherent pulse train signals in narrowband non-Gaussian noise is investigated. The locally optimum (LO) detector structures are developed and found to be in the form of incorporating a locally optimum zero-memory nonlinearity (LOZNL) into the Neyman-Pearson optimum detector for narrowband Gaussian noise. Many practical detectors belong in the same class of structures with the LO detector. The expressions for the efficacies of the detectors are derived. In particular, Weibull and log-normal noise models are considered. The LOZNL's, and the efficacies of the detectors are given, and numerical results are graphically presented. It is shown that, in the sense of the Pitman asymptotic relative efficiency (ARE), the asymptotic performance of many detectors whose nonlinearity can more effectively suppress the tail of the noise envelope distribution is apparently better than that of the Neyman-Pearson optimum detector for narrowband Gaussian noise.

I. Introduction

In the field of the theory of detection in narrowband non-Gaussian noise, papers [1—4] have treated the locally optimum (LO) detection of known signals, paper [5] has dealt with the asymptotic optimum (AO) detection of deterministic and quasi-deterministic signals. But the LO detection of coherent pulse train with unknown initial phase and incoherent pulse train signals has not yet been investigated. As regards the applications, the study of the detection in Weibull and log-normal noises, which are often considered as the models of radar clutter, was mainly concerned with the calculation of small sample performances, and only a few papers [6—7] gave the asymptotic performances of some detectors in log-normal noise. The problem of the optimum detection in the two classes of noises has not yet been considered, except that papers [8] and [9] have determined the Chernoff bound of optimum performance.

In this paper the concept of LO detector [10—11] is applied to the discrete-time detection of coherent and incoherent pulse train signals to develop LO detector structures for these signals. Similarity as well as difference in structure among practical detectors and the LO detector is pointed out. The expressions for their asymptotic performance are derived. In particular, the detection in Weibull and log-normal noises is investigated. The locally optimum zero-memory nonlinearities (LOZNL's), and the efficacies of the detectors in the two classes of noises are given. The numerical results are presented graphically for the asymptotic relative efficiencies (ARE's) of the detectors.
II. Locally Optimum Detector Structure

Consider the following hypothesis testing problem:

\[ H_0, R = \hat{N}, \]
\[ H_1, R = a \hat{S} + \hat{N}, \]

where \( R = (\hat{r}_1, \hat{r}_2, \ldots, \hat{r}_M) \), \( N = (\hat{n}_1, \hat{n}_2, \ldots, \hat{n}_M) \) and \( a \hat{S} = (\hat{s}_1, \hat{s}_2, \ldots, \hat{s}_M) \) are the complex envelope sample vectors of the received narrowband waveform, noise and signal respectively, \( \hat{r}_i = z_i e^{i \phi_i}, i = 1, 2, \ldots, M \), \( z_i \) and \( \phi_i \) are the ith sample of the received waveform envelope and phase respectively, \( a \) represents the average voltage (signal-to-noise ratio, SNR), and \( \hat{s}_i = s_i e^{i \theta_i} \), the ith sample of the signal complex envelope normalized with respect to \( a, s_i \) and \( \theta_i \) are the ith sample of the envelope and phase respectively. Suppose the signal and noise have an equal and known carrier frequency. We assume that the noise envelope and phase are statistically independent, the noise phase is uniformly distributed in \([0, 2\pi]\) and the noise samples \( \{\hat{n}_i, i = 1, 2, \ldots, M\} \) are independent and identically distributed with an envelope pdf expressed as \( p_0(z) \).

When \( H_0 \) is true, the joint pdf of \( z_i \) and \( \phi_i \) is

\[ f_0(z_i, \phi_i) = \frac{1}{2\pi} p_0(z_i), \quad z_i > 0, \quad 0 < \phi_i < 2\pi. \]  

Considering the real and imaginary parts of \( \hat{r}_i \) and transforming variables, we obtain the conditional joint pdf of \( z_i \) and \( \phi_i \) under \( H_1 \) as

\[ f_1(z_i, \phi_i | \theta_i) = \frac{z_i p_0(\sqrt{z_i^2 - 2as_i z_i \cos (\phi_i - \theta_i) + a^2 s_i^2})}{2\pi \sqrt{z_i^2 - 2as_i z_i \cos (\phi_i - \theta_i) + a^2 s_i^2}}, \quad z_i > 0, \quad 0 < \phi_i < 2\pi. \]  

In case of coherent pulse train signals with known initial phase, \( s_i \) and \( \theta_i \) are known, and the likelihood ratio is

\[ A(\vec{R}) = \frac{\prod_{i=1}^{M} f_1(z_i, \phi_i | \theta_i)}{\prod_{i=1}^{M} f_0(z_i, \phi_i)}. \]

This is a case of coherent reception and the LO detector structure is determined by

\[ \frac{\partial A(\vec{R})}{\partial a} \bigg|_{a=0} \geq T \]

where \( T \) is a decision threshold. From Eq. (3), we obtain

\[ \frac{\partial f_1(z_i, \phi_i | \theta_i)}{\partial a} \bigg|_{a=0} = \frac{s_i \cos(\phi_i - \theta_i)}{2\pi} \left[ \frac{p_0(z_i)}{z_i} - \frac{p_0(z_i)}{z_i} \right]. \]  

Using Eq. (6), we obtain from Eq. (4)

\[ \frac{\partial A(\vec{R})}{\partial a} \bigg|_{a=0} = \text{Re} \left\{ \sum_{i=1}^{M} g_{c,i,o}(z_i) e^{i \phi_i \delta_i^2} \right\}, \]

where

\[ g_{c,i,o}(z) = \frac{1}{z} \frac{p_0'(z)}{p_0(z)}. \]