THE ANALYSIS METHOD OF THE SETS OF BRANCHES BASED ON INDEPENDENT LOOPS IN THE ELECTRIC NETWORK

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Abstract The matrix $D$ describing relations of the loops to the nodes in the graph and also the sets of branches based on the independent loops and their matrix $Q$ are defined. The theorem in which the product of the loop-node matrix $D$ multiplied by the incidence matrix $A_*$ is equal to matrix $Q$ is put forward and proved. The admittance matrix $Y_v$ of the sets of the branches is defined and it is assumed that the vector $V_v$ of voltage of the sets of branches to be a calculative quantity. The equation of the sets of branches is derived and the analysis method of the sets of branches based on the independent loops in the electric network is presented.

Key words Electric network; Matrix; Loop branch-set; Loop-node matrix

I. Introduction

It is necessary that the graph structure is described from various aspects in order to study the graph theory. In this respect, many satisfactory successes [1-4] have been achieved. At present matrix description includes incidence matrix $A$, loop matrix $B$ and cut-set matrix $Q$. These matrices were used to describe the graph structure from respective sides with the incidence of all branches in the graph. The branch is an essential factor and the node is another. According to formal logic, naturally, it is assumed that the graph structure can be described equally well from respective sides with the incidence of all nodes in the graph, and various practical applications of this theory are still being studied. Therefore, in this paper, the loop-node matrix is defined first, and the analysis method of sets of branches based on the independent loops in the electric network is presented.

II. The Loop-Node Incidence Matrix $D$, the Product of the Matrix $D$ and the Matrix $A_*$, and the Loop-Branch-Set Matrix $Q$

If $n$ is the number of vertices and $b$ the number of edges in the graph, we may define the loop-node incidence matrix as follows:

$$D = [d_{ij}]_{1 \times n}$$  \hspace{1cm} (1)

where

$$d_{ij} = \begin{cases} 1, & \text{if the } i\text{th loop includes } j\text{th node} \\ 0, & \text{otherwise} \end{cases}$$
the number of independent loops in the graph, \(1 \leq i \leq l, 1 \leq j \leq n\).

As shown in Fig. 1, matrices \(A_*, D\) can be obtained, by multiplying the matrix \(D\) to the matrix \(A_*\), the product matrix \(Q\) will be given. The rows in the matrix \(Q\) correspond to the independent loops chosen in the figure, and the columns - to the all of the branches.

\[
D = \begin{bmatrix}
1 & 1 & 0 & 1 & 0 \\
1 & 1 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 \\
0 & 1 & 1 & 1 & 0 \\
0 & 1 & 0 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 \\
\end{bmatrix}
\]

\[
Q = DA_* = \begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & -1 & 0 & 0 & 1 & 1 & 0 \\
-1 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 1 & 1 \\
0 & 0 & 0 & -1 & 0 & -1 & 0 & -1 & -1 & -1 \\
\end{bmatrix}
\]

From carefully observing the matrix \(Q\), it is easier to discover that all the branches marked "1" in the row of the matrix \(Q\) correspond to all the forked branches connected with the circuit corresponding to this row. The direction of the forked branch corresponding to "+1" leads to the outside of the vertex of the loop, "-1" to the inside of the vertex of the loop and "0" corresponding to the branch not connected with the forked branch. Therefore, the product matrix \(Q\) shows that all of the branches in the graph fork from all independent loops and so it is the incidence matrix between every independent loop and the forked branches from it. It is also called a loop branch-set matrix for short.

Now, let the concepts about the loop branch-set matrix and so on be defined in a very general sense. Provided that the electric network is a connected graph including \(n\) vertices and \(b\) branches, the following definition will be given: