Pion-Nucleon Scattering by Variational Method.

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Summary. — $S$ wave phase shifts in pion nucleon scattering are calculated in a non-relativistic approximation of the $PS$ ($PN$) theory with a cut-off at the nucleon mass. The variational procedure proposed by CINI and FUBINI is applied; charge renormalization is performed in a manner similar to that used by DESER, THIRRING and GOLDBERGER. For pion energy sufficiently high, the phase shifts found have the correct sign and qualitative energy dependence; however, the magnitudes are much too large.

1. — Introduction.

The extended source approximation gives sensible results for the $P$ wave phase shifts in pion nucleon scattering. Both a Tamm-Dancoff solution (1) and a calculation (2) based on the variational method of CINI and FUBINI (3) give $\alpha_{33}$ ($T=3/2, J=3/2$) as the dominant phase shift, as seems strongly indicated by the experimental data. Using slightly different values of the coupling constant and cut-off momentum, these calculations give a good quantitative fit to the $P$ wave part of the Glicksman solution (4) for the phase shifts.

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(1) G. F. CHEW: Phys. Rev., 89, 591 (1953), and other papers in press.
(3) M. CINI and S. FUBINI: Nuovo Cimento, 11, 142 (1954), referred to as CF.
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The $P$ wave calculations use as an interaction Hamiltonian

$$H_P = \frac{g}{2M} \int q(x) \sigma \cdot \nabla \tau_i \Phi_i d^3x. $$

(1) follows from the relativistic $PS$ ($PS$) Hamiltonian

$$H = ig \int \bar{\psi} \gamma_5 \tau_i \Phi_i \psi d^3x, $$

(2)

if one considers only positive energy states and neglects the recoil of the nucleon, putting its energy always equal to $M$. The effect of the recoil is roughly compensated by introducing a cut-off in the momentum integrals; the cut-off function is the Fourier transform of the source density $q(x)$.

Applying a similar approximation to the processes which contain nucleon pairs in intermediate states, one can calculate the $S$ wave scattering. However, one obtains in this way a scattering independent of isotopic spin, in disagreement with the experimental data. Such a procedure corresponds to using in a fixed source theory the so-called "pair" Hamiltonian

$$H_s = \frac{g^2}{2M} \int \Phi_i^2 q(x) d^3x, $$

(3)

which is the leading term giving rise to an $S$ state interaction in the expansion of the Dyson-transformed $\gamma_5$ Hamiltonian as given by DRELL and HENLEY (5). The gradient coupling term (1) is the lowest order term in this expansion.

In order to obtain an isotopic spin-dependent scattering, one might consider including, for example, the next term in the expansion of DRELL and HENLEY,

$$H_c = \left( \frac{g}{2M} \right)^2 \int \tau \cdot \Phi \cdot \Pi q(x) d^3x. $$

(4)

However, the use of $H_s + H_c$ as a Hamiltonian is an inconsistent procedure in all orders after the lowest. Comparison of the fourth order matrix elements obtained in this way with those of the correct relativistic Hamiltonian (2) shows that terms are neglected of precisely the same nature as those retained (6).

In this work we calculate the $S$ wave scattering from the Hamiltonian (2) and pass to the non-relativistic limit retaining the first two orders in the expansion of the nucleon energy. It is still necessary, of course, to introduce

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