Angular Distribution of Deuterons from $^9\text{Be}(p, d)^{8}\text{Be}$.

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In the last years Butler's theory of stripping is used successfully in nuclear spectroscopy. Recently the reversed-pickup process was investigated \cite{1, 2} with use of the reciprocity theorem. On applying Born's approximation in the present paper we obtain the differential cross-section for the $^9\text{Be}(p, d)^{8}\text{Be}$ reaction. This method was used for the stripping by Bathia et al. \cite{4}. For the $^8\text{Be}$ nucleus we take the two body (nuclear shell) model. In the calculation we neglect the neutron-$^8\text{Be}$ interaction compared with the neutron-incident proton interaction. We do so, because the n-p interaction is responsible for the process investigated and because in our case this interaction is larger than the n-$^8\text{Be}$ one \cite{3}. The p-$^8\text{Be}$ interaction is neglected and particularly the Coulomb effect is not taken into consideration.

The differential cross-section in the center of mass system under our assumption is given by Eq. (1):

\begin{equation}
\frac{d\sigma}{d\Omega} = \frac{1}{2\pi \hbar^2} M_p^* M_d^* (k_d/k_p) \frac{1}{8} \sum_{m'_i \mu'\sigma'_d} \int \frac{d\hat{r}_d}{d\xi} \frac{d\hat{\rho} d\sigma_d d\sigma_p \chi^*_d(\rho \sigma_n \sigma_p)}{\chi^*_d(\rho \sigma_n \sigma_p)} \exp \left[ -i k_d \hat{r}_d \right] V_{p, n} \chi_d(\hat{r}_n \sigma_n) \chi_p(\sigma_p) \exp \left[ i k_p \hat{r}_p \right] ^2,
\end{equation}

where $M_p^*$, $M_d^*$ are respectively the proton and deuteron reduced mass; $\hbar k_p$, $\hbar k_d$ are respectively the proton and deuteron momenta (in the c.m. system); $\chi_d$, $\chi_p$, $\chi_n$, $\chi_\sigma$ are respectively the deuteron, $^8\text{Be}$, proton and $^8\text{Be}$ internal wave functions; $\rho$ is vector from neutron to proton, $r_n$ vector from $^8\text{Be}$ to neutron, $r_p$ vector from $^8\text{Be}$ to proton; $\sigma_n$, $\sigma_p$ are respectively neutron and proton spin coordinates, $\xi$ internal $^8\text{Be}$ coordinates; $m_i$, $\mu_p$, $\mu_d$ are respectively $^8\text{Be}$, proton and deuteron magnetic numbers; $V_d$ is the deuteron potential.

\cite{1} J. A. Harvey: Phys. Rev., 82, 298 (1951).
\cite{6} A. B. Bathia, Kun Huang, R. Huby and H. C. Newns: Phil. Mag., 43, 485 (1952).
\cite{7} Thus we neglect the interaction, which plays the most important role in the reciprocity theorem method.

\cite{8} For spinless particles and rigid center of force one can obtain the analogue of Eq. (1) from the equation (18) given by E. Clementel: Nuovo Cimento, 11, 412 (1954).
For the n-^Be interaction we take the spherical well of depth $V_0 = 12.09 \text{ MeV}$ and radius $r_0 = 5 \cdot 10^{-3} \text{ cm}$. Then we have $\chi_l = \chi_l(\xi)\Phi_m(\xi\sigma,
abla\sigma)\mathcal{R}(r_n)$; $\Phi_m\mathcal{R}$ is the wave function for the $P_1$ state given e.g. by Uberall (*) . For $V_a$ we take the well of depth $U_o = 21 \text{ MeV}$ and radius $a = 2.82 \cdot 10^{-3} \text{ cm}$. Then $\chi_a$ is the $S$-ground state wave function for the above $V_a$ ($\chi_a = S_m(\sigma\sigma_a)\psi(q)$).

On inserting these functions into (1) and taking in the expansion of $\exp[-i\hbar \mathcal{R}]$ in a series of spherical waves the only term to give the non-vanishing contribution to the integral (with $l=1$) and on performing all the integrations and summations we finally obtain:

\begin{equation}
\frac{d\sigma_a}{d\Omega_a} = (1/2\pi\hbar^2)^3 M_a^* M_a(k_d/k_a)\frac{3\pi G(K)}{\mathcal{I}_0}(k_a)^2.
\end{equation}

\begin{equation}
G(K) = \frac{4\pi}{K} \int_0^\infty dq \cdot q V_a(q)\psi(q) \sin Kq =
(2\pi A U_0/K) \{\sin (\kappa+K)a/(\kappa+K) - \sin (\kappa-K)a/(\kappa-K)\},
\end{equation}

where \( A = \sqrt{2\pi(1+\alpha)} \), \( \alpha = \sqrt{M(U_0 - \epsilon_a)/\hbar} \) (\( \alpha = \sqrt{M\epsilon_a/\hbar} \), \( \epsilon_a \) = deuteron binding energy, \( M \) = nucleon mass).

\begin{equation}
\mathcal{I}_0(k) = \sqrt{\pi/2} \int_0^\infty dr_n r_n^2 \mathcal{R}(r_n) J_\frac{3}{2}(kr_n)/\sqrt{k r_n},
\end{equation}

appears identical with the integral calculated by Uberall (*) (Eq. (13)) in the theory of the photoeffect of $^4\text{Be}$ at large energies. \( K \) and \( k \) are respectively given by:

\begin{equation}
K^2 = \frac{1}{4} k_d^2 + k_p^2 - k_d k_p \cos \theta_v, \quad k^2 = k_d^2 + \frac{64}{81} k_p^2 - \frac{16}{9} k_d k_p \cos \theta_v,
\end{equation}

where \( \theta_v \) is the scattering angle in the c.m. system.

Fig. 1 represents the relative values of the differential cross-section in the laboratory frame ($d\sigma_L/d\Omega_L$) compared with the experimental data given by Cohen (\(^2\)) (22 MeV protons) and Harvey (\(^4\)) (8, 7, 6, 5 MeV, average 6.5 MeV). The theoretical angular distribution for 22 MeV protons sinks to zero value for large angles faster than the experimental angular distribution (this is a common feature of stripping reactions). Unfortunately there are no experimental data for small angles for 22 MeV. The uncertain present data for 5-8 MeV protons give no possibility of precise comparison of our results with experiment.

The absolute values of $d\sigma_L/d\Omega_L$ for the angles $\theta_L$: 0° and 20° resulting from (2) are respectively for 22 MeV protons: 1.4, 0.5 barn/steradian and for 6.5 MeV protons: 2.6, 2.0 barn/steradian. The absolute value of $d\sigma_L/d\Omega_L$ resulting from Harvey’s experiments seems to be about 0.024 barn/steradian. However this discrepancy

(*) H. Uberall: Zeits. f. Naturforsch., 8a, 142 (1943).