The very illuminating discussion by Bohr and Rosenfeld (1) on the problem of field quantities measurability in Quantum-Electrodynamics, gives results which may be considered hardly open to any doubt (2) on all the aspects of the question considered by the Authors.

There is however one point which may have some interest in the discussion of the quantum electrodynamics formalism from a physical point of view, and which has not been considered by Bohr and Rosenfeld.

The point is the following: how is it,

(1) N. BOHR and L. ROSENFELD: *Kgl. Danske Vid. Selsk. Math. Fys.*, 12, n. 8 (1933); *Phys. Rev.*, 59, 896 (1911). (2) In a well known paper HALPERN and JOHNSON (O. HALPERN and M. H. JOHNSON: *Phys. Rev.*, 59, 896 (1911)) have stated that admitting the existence of arbitrarily constituted test bodies, the accuracy of the measurement (of field strengths) still cannot exceed certain limits which are mainly defined by the wave-length of the field and the spatial and temporal domain of measurement.

The argument is that the electric charge density of the test bodies cannot be thought arbitrarily high because of the vacuum disruptive currents, which take place for the creation of (real) electron positron pairs.

Recently Prof. Halpern has further emphasized that due to the mentioned circumstance, when the charge density of the test body should be too high, it is even impossible to prepare the device for Bohr-Rosenfeld's ideal field measurements (private communication).

Now it seems that this kind of argument cannot possibly be completely correct, merely for relativistic reasons. Such notions as the wave length of the field and so on, are not indeed relativistic invariant, and if therefore we change frame of reference we can pass from a wave length as short as we like to a wave lengths as long as we like in the same physical circumstances (at least in the case of progressive waves). The same may be stated for charge density. We can realize a charge density as high as we like taking a body charged at rest with a moderate charge density and moving it sufficiently fast. It should however be remarked that we have then, together with the charge density, an electric current different from zero.

This last point gives us the key for the solution of the supposed paradox concerning Bohr-Rosenfeld's measurement devices.

One may think in effect of compensating the vacuum disruptive currents, by some suitable low-resistance source of e.m. force between the charged test-bodies of Bohr-Rosenfeld's devices and the neutralising bodies.

The essential point here is that, as long as the dimension and the density of the charge of the test body are finite, the intensity of the
possible to define states of the e.m. field by measurements, or, which is the same, how is it possible to perform complete sets of measurements of the field quantities?

Even in the classical theory this point is not trivial. Considering in effect the field like a mechanical system with an infinite number of degrees of freedom, it would appear to be necessary in general a set of an infinite number of measurements for defining exactly and completely the state of the system in the classical sense, and this of course would be an impossibility.

In Quantum Theory however this difficulty luckily doesn't arise, but, with all that, a thorough discussion of the question taking into account the interaction with the positron-electron field, would not be too easy.

In this letter I only like to consider the case of the electromagnetic field without the interaction with the electron-positron field.

First it should be noted that, in order to have the possibility of defining the state of the system by a convenient set of measurements, we cannot take in consideration the case of the field in the infinite space, but we have to think of a bounded region of the space only, like for instance that inclosed in a box with perfectly reflecting walls (3).

Vacuum disruptive currents is not infinite; this circumstance may reasonably be suspected from the previous relativistic argument and may be confirmed by a more direct argument. A compensation of the vacuum disruptive currents is therefore possible and no real difficulty arises in preparing Bohr-Rosenfeld's measurements.

(3) This is a trivial example of the limitation that from physical requirements may be put on the mathematical formalism in quantum electrodynamics. It is a known thing that from a mathematical point of view it is essentially different to consider the Hohraum infinite from the beginning, or finite, and then, if it is the case, to make its volume going to infinite. (See for instance S. T. MA: Phys. Rev., 87, 632 (1952); B. Ferretti: Nuovo Cimento, 10, 1079 (1953).

It is quite easy then to see that «closed sets» of measurements exist which allow to define completely (in quantum mechanical sense) the state of the system. The «vacuum state» for instance may be defined by a weighing operation of the field containing box. It may be remarked however that this kind of operation is not a measurements of the field strengths but merely of the total energy of the field.

It is fitting to point out immediately that it is impossible to put up a complete set of measurements by measuring field strengths only. This circumstance however does not forbid the possibility of performing a complete set of measurements by measuring field strenghts and some other quantity togethether.

One convenient manner of carrying on the analysis of the present problem is to consider for instance the normal modes of vibration of the field containing box.

If \( k \) are the propagation vectors and \( u_{ik}, v_{ik} \) are the polarization versors defining this normal modes, \((u_{ik} \parallel \text{electric field and } v_{ik} \parallel \text{the magnetic field})\) and if \( E \) and \( H \) are the electric and magnetic fields, putting

\[
\begin{align*}
H'_{ik} &= \int_{\Omega} a \times H \cos k \cdot r \, dr, \\
H''_{ik} &= \int_{\Omega} a \times H \sin k \cdot r \, dr,
\end{align*}
\]

and so on (where the integration is extended to the volume of the field containing box and \( a \) and \( b \) are given versors) we have, as well known, the

\[\text{App. I}\). It seems here clearly indicated that when a difference arises in the two treatments the second one should be preferred for physical reasons.