Theory of Scattering in the Quantized Field
and Low-Chew-Wick’s Formalism.

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Summary. — The Lippmann-Schwinger equation is studied from the viewpoint of the quantized field theory. The general prescription for separating the “incident physical state” is given by introducing the modified contraction technique. The Low-Chew-Wick formalism is derived from the L.S. equation by virtue of our contraction technique which makes the former more easy to understand.

Introduction.

In applying the Lippmann-Schwinger equation (*) (1)

\[
\Psi_{q}^{(\pm)} = \varphi_{q} + \frac{1}{E_{q} - H_{0} \mp i\epsilon} H_{1} \Psi_{q}^{(\pm)},
\]

to the scattering problems in the quantized field theory, there occur some cumbersome problems.

1) In the particle dynamics the validity of the L.S.-equation depends on the fact that the interaction-Hamiltonian \(H_{1}\) induces only a negligibly small shift of the energy levels in the continuous spectrum. In the field theory, however, we encounter the difficulty of the infinite shifts of the energy levels which is usually overcome by the renormalization procedure (2). Namely, the

(*) Hereafter, this equation is referred to as L.S.-equation.
unrenormalized Hamiltonian
\[
H = H'_0 + H'_1
\]
is replaced by the renormalized ones
\[
H_o = H'_0 + \sum_a \langle \varphi_a | \Delta E_a | \varphi_a \rangle , \tag{2a}
\]
and
\[
H_i = H'_1 - \sum_a \langle \varphi_a | \Delta E_a | \varphi_a \rangle , \tag{2b}
\]
where $\varphi_a$ is the eigenfunction of the renormalized free Hamiltonian $H_0$ and $\Delta E_a$ is the level-shift of this state and is so determined as to make spectra of $H$ and $H_0$ coincide with each other in the continuous region.

2) As DeWitt (*) pointed out, in the field theory, the state vector $\Psi_s$ in (1) is not normalized to unity owing to the fact that the interaction $H_i$ is an integral over the whole space, even though $\varphi_0$ has been normalized to unity.

3) Contrary to the scattering cases in the particle dynamics, the incident "physical state" is described not only by the $\varphi_a$ but also by some parts of the second term of Eq. (1), because the former represents the incident "bare-state", while the latter the incident nucleons dressed with many virtual field quanta. For the calculation of the scattering cross-section the dressed states mentioned above should be separated from the second term in Eq. (1). It will be seen that this separation will allow us to normalize the $\Psi_s$ in the field theory.

The main purpose of the present article is to give the general prescription for this kind of separation, by introducing some contraction-techniques which are similar to that device by Wick (4) in the covariant field theory. In the present paper, for simplicity, the case of meson-nucleon scattering is exclusively studied, although our method is, of course, applicable to more complicated problems provided that the formation of nucleon-pairs is excluded.

In the case of the meson-nucleon scattering, we shall be able to give a field theoretical foundation to Wick's (4) formulation concerning Low-Chew theory (5) (6) from our view point. This is the second purpose of the present paper.

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(*) In what follows, the abbreviation L.C.W. is used.