On a Regular Field Theory.

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Summary. — A regular field theory is constructed in a finite space-time domain Ω restricted by two space-like hypersurfaces. The field equations are integral equations with regular kernels. The solutions are integer functions of the coupling parameter. Integral conservation laws are inferred. In order to secure the correspondence with the traditional field theory the form factor (introduced to regularize the kernels) must depend on the domain Ω. The integral field eq-s are quantized directly, without any reference to the canonical formalism. The Schrödinger-Tomonaga eq. does not exist but the formalism yields unambiguous results concerning the probability amplitudes connecting the measurements on the two hypersurfaces. The author represents the opinion that field quantization has nothing to do with the convergence difficulties. They disappear in the case of a finite domain and of regular kernels.

Introduction and summary.

Although the convergence difficulties inherent in the relativistic quantum field theory are known and extensively discussed for more than twenty years, their origin is not yet fully realized, and there is a discrepancy of opinions in this respect among physicists. Some of the authors believe that the reason of the difficulties is physical in character and, in order to get a consistent formalism, new physical assumptions are necessary (for example the introduction of a fundamental length λ). However, the introduction of a non-local point of view (necessarily connected with a fundamental length) was regarded, for many years, as impossible since the non-local fields resist quantization (in the traditional meaning of this word). This led some physicists to inves-
tigate the question of adequacy of the formalism of quantization which seems to be too tight to include more general types of fields and interactions. Among many attempts we note Heisenberg's $S$-matrix theory amounting to a refusal of the description of events in space-time.

Several authors represent an opposite viewpoint: they seem to believe that the usual quantum theory of localizable fields is consistent while the difficulties arise simply from the inadequacy of the traditional methods of solving approximatively the field equations (or solving the Schrödinger equation) by means of power series expansion in terms of the coupling constant. These authors (Schwinger, Dyson, Feynman and others) try to improve the technique of the power expansion method by means of a procedure of renormalization of physical constants appearing in the field equations. They achieved a quantitative success in the electrodynamics but have not proved whether their procedures are unambiguous, and whether the series converges after the renormalizations. We note that the renormalization program is not practicable in the case of most of meson theories.

From the mathematical viewpoint the possible reasons for the convergence difficulties encountered in the traditional field theories are threefold: (i) a singular (infinite) space time domain in which physicists used to investigate their field-theoretical problems, (ii) a singular character of the kernels of the integral field equations (which formally are equivalent to the traditional differential field equations), (iii) a singular character of the commutation relations between the field components. In most of the contemporary investigations these three possible reasons of difficulties appear hopelessly intermingled.

The first possible reason of difficulties may be removed simply by taking a well defined finite domain $\Omega$ where the system of fields undergoes an evolution starting with proper initial conditions.

The singular character of the kernels may be avoided by introducing a new physical assumption: a non-local interaction. This may be achieved by introducing a relativistic form factor discussed previously by the author (1) (2), and by Kristensen and Möller (3). The existence and uniqueness of the solutions under the assumption of regular initial conditions is then easily demonstrated by means of the method due to Picard.

Conservation laws applying to the surfaces enclosing the domain $\Omega$ are inferred. With a particular choice of the type of the form factor the formalism satisfies the principle of correspondence: the field equations and the expressions for the conserved quantities (charge, energy, momentum, and angular momentum) go over into the local ones in the limit $\lambda \rightarrow 0$.

(2) J. Rayski: Phil. Mag., 42, 1289 (1951).