Analysis of the New Conservation Law in Electromagnetic Theory.

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Summary. — The new conserved tensor in electromagnetic theory introduced by Lipkin is shown to be a member of an infinite sequence of moments constructed from the wave vector. The physical interpretation is discussed.

1. — Introduction.

In a recent paper (1) on the classical electromagnetic field, Lipkin exhibits a symmetric traceless tensor in the form of the integral of a conserved density over a spacelike surface. The physical interpretation is left open, apart from an indication that the sign of the quantities depends on the sense of circular polarization (in quantal terms, on the helicity).

In the present paper, this tensor and the energy-momentum vector are displayed as integrals over invariant amplitudes for components of definite polarization and wave-vector, and shown to be members of an infinite sequence of wave-vector «moments»; it is conjectured that half of these «moments» are susceptible of display as integrals of conserved densities over spacelike surfaces in space-time.

2. — Formalism.

In this Section, the solution $E$ and $H$ of Maxwell’s equations without sources are expressed in terms of invariant amplitudes $q^\pm(k)$ for right and left circular polarization.

larly polarized components of wave-vector \( k \); vector and pseudovector potentials are also introduced.

Suppose that we have an electromagnetic disturbance of the vacuum of finite total energy

\[
P_0 = \frac{1}{2} \int (E^2 + H^2) \, d^3r < \infty.
\]

By Plancherel's theorem, we can introduce Fourier transforms

\[
E^*(k, t) = (2\pi)^{-\frac{3}{2}} \int E(r, t) \exp[-i k \cdot r] \, d^3r
\]

and

\[
H^*(k, t) = (2\pi)^{-\frac{3}{2}} \int H(r, t) \exp[-i k \cdot r] \, d^3r.
\]

From Maxwell's equations, these can be written

\[
E^*(k, t) = k^{-\frac{1}{2}} [e(k) \exp[-ikt] + e^*(-k) \exp[ikt]],
\]

and

\[
H^*(k, t) = k^{-\frac{1}{2}} [h(k) \exp[-ikt] + h^*(-k) \exp[ikt]],
\]

where \( k = |k| \) (with \( c = 1 \)) and \( e, h \) satisfy

\[
k \cdot e = 0, \quad k \cdot h = 0, \quad k \times e = kh, \quad k \times h = -ke.
\]

It is convenient to introduce standard solutions \( u^\pm(k) \), which in the quantized theory would be the wave functions for right and left circularly polarized photons. They are defined apart from an arbitrary phase by

\[
\text{ik} \times u^\pm(h) = \pm u^\pm(k),
\]

with the invariant normalization

\[
[u^\pm(k)]^* \cdot u^\pm(k) = k^2.
\]

We can now expand

\[
e(k) \pm ih(k) = q^\pm(k) u^\pm(k),
\]

where the amplitudes \( q^\pm(k) \) are subject to no restriction other than the trans-