The S-Wave in Pion-Nucleon Scattering. (*)

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Summary. — A calculation is made of the two S-phase shifts of pion-nucleon scattering corresponding to isotopic spin states $T=1/2$ and $T=3/2$, using the Tamm-Dancoff method and an extended source model for the nucleon. The magnitude and energy variation of the $\alpha_3$ phase-shift agree reasonably well with experiment. The sign of the $\alpha_1$ phase-shift is correct, but its magnitude is completely wrong. It is likely, however, that $\alpha_1$ is the S-phase shift which will be seriously altered by taking account of renormalization effects in a more rigorous treatment of the problem.

Recent experiments on pion-nucleon scattering, particularly at Rochester (1) and Columbia (2) in the 35-65 MeV region, have fixed (3) the absolute signs as well as the magnitudes of $\alpha_3$ and $\alpha_1$, the S-phase shifts corresponding to the isotopic spin $T=3/2$ and $T=1/2$ states respectively. It turns out that $\alpha_3$ is negative and changes by a factor of 3 (from $-2^\circ$ at 35 MeV to $-6^\circ$ at 65 MeV), whereas $\alpha_1$ is positive and remains essentially constant (at about $10^\circ$). When these results are considered in conjunction with the results of the

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(3) The Fermi-Yang duality still remains, of course; however, the S-phase shifts are chosen so that the $\alpha_{33}$ phase shift is positive and the largest of the P-phase shifts (cf. Proc. Rochester Conference on High Energy Physics, Interscience Publishers, 1953).
Chicago experiments at higher energies \(^{(2a)}\), it becomes clear that \(\alpha_3\) is a rapidly varying negative, and \(\alpha_1\), a slowly varying positive function of the energy from 35 to at least 150 MeV. The opposite signs of the two phase shifts and their completely different energy dependence can easily be explained by a phenomenological theory \(^{(4)}\) which postulates an attractive potential with a repulsive core for the \(\alpha_3\) phase shift and a pure attractive potential for \(\alpha_1\). However, in this note, we report on an attempt to determine whether the renormalizable PS(PS) theory (pseudoscalar theory with pseudoscalar coupling) can explain even qualitatively the signs and energy dependence of the S-phase shifts.

At first sight, the PS(PS) theory does not offer a promising explanation of the S-phase shifts. Since we are interested in pion-nucleon scattering at non-relativistic energies (for the nucleon), we can examine the two leading terms in the canonically transformed Hamiltonian which are usually considered \(^{(4)}\), namely:

\[
\frac{G\mu\nu^*}{2M} \sigma\cdot \nabla \varphi \psi + \frac{G^2\mu}{2M} \psi^* \varphi^2 \psi,
\]

where \(\psi\) is the nucleon wave function, \(\varphi\) the pion wave function, \(M\) and \(\mu\) are the respective masses, \(G\) is the coupling constant and \(\sigma\) and \(\tau\) are the spin and isotopic spin operators of the nucleon. The first term in (1) gives the \(P\) wave scattering and the second term (core term) gives \(S\) scattering; however, it is clear that the repulsive core term yields the same negative sign and magnitude for \(\alpha_3\) and \(\alpha_1\) in all approximations — in contradiction to experiment. The next term in the canonically transformed PS(PS) Hamiltonian is \(^{(4)}\)

\[
\left(\frac{G\mu}{2M}\right)^2 \psi^* \tau \cdot \varphi \wedge \pi \psi,
\]

where \(\pi\) is the canonically conjugate momentum to \(\varphi\); (2) represents a coupling between the isotopic spins of the nucleon and the pion (\(\int d\varphi \cdot \pi\) is equal to \(\theta\), the isotopic spin of the pion) and leads immediately, in Born approximation, to a negative \(\alpha_3\) but to an \(\alpha_1 = -2\alpha_3\). If one now recalls that an improved Tamm-Dancoff calculation \(^{(7)}\) of the \(P\)-phase shifts changes the Born approximation results in the direction of enhancing the attractive \(\alpha_3\).


