On the Potential Description of the VN-NN\(\theta\) Sector of the Lee Model.

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Summary. — The VN-NN\(\theta\) sector of the Lee model is studied by making use of the clothing-transformation technique. The space is decomposed in two uncoupled orthogonal subspaces and the Hamiltonian expressed in the clothed basis is the sum of the VN and NN\(\theta\) interactions. We obtain all the elastically equivalent potentials which give the correct on-shell \(NN\) scattering amplitude and find that they are necessarily three-body potentials. The limit of infinite coupling constant and infinite internal energy of the V-particle is also discussed.

1. — Introduction.

In connection with the problem of many-body forces in nuclear interactions, it is interesting to study the behaviour of a multiparticle system in the framework of simple field-theoretic models. For instance, the Lee model allows one to calculate the potential energy of a system of \(n\) fixed baryons arising from the exchange of a \(\theta\)-particle (1). In this case the static condition leads to a nonambiguous determination of the multiparticle component in the interaction.

In this paper we shall consider the NV-NN\(\theta\) sector of the Lee model, and study the possibility of describing by means of a potential the scattering of a \(\theta\)-particle by two fixed \(N\). We apply to the sector the technique of the "cloth-

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ing transformation \( T \) \(^{(2)} \), which we have already used for the V-N0 sector \(^{(3)} \). Also in this case we obtain all the energy-independent equivalent potentials, which reproduce the correct on-shell scattering amplitude. These potentials necessarily depend on the relative position of the two N-particles, that is they have a three-body character.

It is well known that, in potential theory, three-body forces may have a rather fictitious meaning. In fact phase-equivalent two-body potentials \(^{(4)} \) give different results in the three-body system. Three-body forces can therefore be introduced in order to account for this difference, which is due to the off-shell behaviour of the two-body scattering amplitude.

In our case we find that none of the possible elastically equivalent N0 potentials can account for the scattering of a 0-particle by two N-particles. This is clearly due to the fact that the field-theoretic off-shell \( t \)-amplitude for the N0 scattering cannot be obtained by any energy-independent potential. So we can say that, in this case, three-body forces arise if we claim to describe a field-theoretic interaction by means of a potential theory.

As we have already noted in I, the clothing technique is the most appropriate for studying the \( \epsilon \) bound state \( \epsilon \) limit. It is well known that in this limit the \( \theta N \) interaction is described completely by a simple separable potential, even in sectors containing more than one N-particle. We find however that this interaction does not exhaust the model. In fact the \( \epsilon \) clothed \( \epsilon \) V-particle tends to the bare one and interacts with the N-particle through a simple exchange potential.

2. – The model.

We first recall the results of the VN sector of the Lee model. For our purposes it is more appropriate to consider the nonrelativistic version of the model, so that we introduce an internal energy for the V-particle and write the Hamiltonian as

\[
\mathcal{H} = U_0 \int \mathbf{d} r \, V^\dagger(\mathbf{r}) \, V(\mathbf{r}) + \int \mathbf{d} \mathbf{k} \, \omega_\mathbf{k} \, \theta^\dagger(\mathbf{k}) \, \theta(\mathbf{k}) + \\
+ \lambda \int \mathbf{d} \mathbf{r} \int \mathbf{d} \mathbf{q} \left[ f(q) \exp \left( -i \mathbf{q} \mathbf{r} \right) N^\dagger(\mathbf{r}) \, V(\mathbf{r}) \theta^\dagger(\mathbf{q}) + \text{h.c.} \right] = \mathcal{H}_0 + \mathcal{H}_I ,
\]

where \( \omega_\mathbf{k} = k^2/2m_0 \).