A Narrow-Resonance Approach to Current Algebra Sum Rules: Meson-Nucleus Low-Energy Scattering (*) (**).

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(ricvuto il 15 Maggio 1973)

Summary. — $SU_3 \otimes SU_3$ chiral algebra and PCAC have been employed to derive pion and kaon scattering amplitudes on nuclei from sum rules of the Fubini-Furlan type. The nuclear states are described as finite systems in a simple, nonrelativistic impulse approximation. The effects of the break down of chiral $SU_3 \otimes SU_3$ symmetry are described in a $(3, \bar{3}^*) + (\bar{3}^*, 3)$ model. The $\pi N$ sigma-term turns out to be about $46 \text{ MeV}$ and we find good agreement between the model by Gell-Mann, Oakes and Renner and both pion and kaon-nucleus data.

1. – Introduction.

In a recent note (1) (to which we shall refer as A in the following) we discussed the derivation of current algebra sum rules (2) and we applied them to meson-baryon low-energy elastic scattering in the limit of infinitely narrow resonances, assuming moreover these resonances to completely saturate the sum rules.

We extend here this approach to the elastic scattering of pions and kaons on nuclei, hoping that the main features of these processes at threshold will be described by such a simple and conventional (*** ) approach as well as are those of meson-baryon elastic scattering.

(*) To speed up publication, the author of this paper has agreed to not receive the proofs for correction.

(**) Work supported in part by Istituto Nazionale di Fisica Nucleare.


(*** ) At least for those particle theorists which are more familiar with the language of Feynman graphs and dispersion relations than with that of configuration space descriptions.
To simplify as far as possible the treatment we shall also adopt, from pioneering works in this field (1), a very simple description of the nucleus, consisting in the nonrelativistic limit of an impulse approximation.

We shall later see that this approach, however naive it may look, gives both qualitative and quantitative predictions verified by the data on meson-nucleus interactions in \( \pi \) and \( K \)-mesic atoms (4,5).

These results can also be considered as corrections to the low-energy theorems of Balachandran, Gundzik, Nicodemi, Tomozawa and Weinberg (4,7); but these corrections, being in general roughly proportional to the mass number of the target, even for pion scattering soon overcome the values obtainable by a naive extension, to \( q^2 = m_i^2 \), of the theorems derived from current algebra and PCAC at \( q^2 = 0 \).

2. – The sum rules.

We derived in \( A \) the two sum rules

\[
(2.1) \quad P \int \xi_{\text{in}}(v, q^2) \delta(q^2 - v^2/p_0^2)(1 - q^2/m_i^2)v^{-2} dv dq^2 = 4\pi Q_i - C_{\text{in}}^{(v)}/m_i^2 p_0 \\
\]

and

\[
(2.2) \quad P \int \xi_{\text{in}}(v, q^2) \delta(q^2 - v^2/p_0^2)(1 - q^2/m_i^2)v^{-1} dv dq^2 = -2\pi p_0 \Sigma_{\text{in}} - C_{\text{in}}^{(v')}/m_i^2 ,
\]

from the chiral-algebra equal-time commutation relations of, respectively, two axial charges and of an axial charge with an axial divergence. Here \( v = q \cdot p \) (\( p \) and \( q \) are the four-momenta carried respectively by the target and the incident meson), and the distribution \( \xi_{\text{in}}(v, q^2) \) is defined as the limit for \( q \to 0 \) of the absorptive part of the correlation function for two axial divergences in the ground state of the target

\[
(2.3) \quad \xi_{\text{in}}(v, q^2) = \lim_{q \to 0} \frac{1}{2} \int d^4 x \exp [i q \cdot x] \langle N, p | [\partial_{\mu} A_{i}^\mu(x), \partial_{\nu} A_{j}^\nu(0)] | N, p \rangle .
\]

The subscript \( i \) runs over the two symbols \( \pi \) and \( K \), indicating the kind

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