Isospin-Variant State Functional for the Yang-Mills Field.

C. A. UZES

Department of Physics and Astronomy, University of Georgia - Athens, Ga.

(riccuvuto il 7 Aprile 1972)

Summary. — The gauge-covariant decomposition of the gauge potentials into their transverse and longitudinal parts is used to construct an isospin-variant "good" state functional for the non-Abelian gauge field.

The canonical gauge and Lorentz-invariant quantization of the Yang-Mills field of Loos (1) was carried out in the Schrödinger representation in which physical states are represented by gauge-invariant "good" functionals of the Yang-Mills spatial potentials. However, at present no gauge-invariant "good" functionals are known which have the property that they transform under a nontrivial representation of the isospin group, although a lengthy mathematical proof has been provided for their existence (2). Such states are required if the quantization is to describe a gauge field whose quanta can carry isospin. The purpose of the present note is to point out that the Treat (3) decomposition of the spatial gauge potentials into gauge-covariantly defined transverse and longitudinal parts can conveniently serve as the basis for the construction of some isospin-variant "good" state functionals.

In the quantization scheme of ref. (1) the Schrödinger representation is employed in the Schrödinger picture. The operators (4)

\[
\pi^\alpha_i(x) = -\frac{i}{\hbar} \frac{\delta}{\delta b^\alpha_i(x)}
\]

(\(\alpha = 0, 1, 2, 3\))


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are canonically conjugate to the gauge potentials \( b_\mu(x) \). Good state functionals \( \psi \) are those which satisfy the constraints

\[
\begin{align*}
\nabla_\alpha \pi^\alpha \psi &= 0 \\
\pi^\alpha \psi &= 0 ,
\end{align*}
\]

\( (\alpha = 1, 2, 3) \),

where

\[
\nabla_\alpha V^\alpha = \varkappa_\alpha V^\alpha - \epsilon_{m\alpha} b^m_\alpha V^\alpha
\]
denotes the gauge-covariant derivative of the internal vector field \( V^\alpha(x) \). The constraints (2) ensure that the quantized Yang-Mills field equation acting on good states has the same form as the classical field equation.

One must take care at the outset to distinguish between isospin and gauge transformations. Under infinitesimal transformations of either type the gauge potentials suffer the change

\[
\delta b^\alpha_\mu = -\nabla_\alpha \eta^\mu ,
\]

where \( \eta^\mu \) is the infinitesimal parameter defining the transformation. If one writes

\[
\lim_{|x| \to \infty} \eta^\mu = e^\mu + O\left(\frac{1}{|x|}\right) \quad \quad (e^\mu = \text{const}) ,
\]

then \( \eta^\mu \) is said to define an infinitesimal isospin transformation if \( e^\mu \neq 0 \), a gauge transformation if \( e^\mu = 0 \).

With the above definitions Loos (1) has shown that "good" states are represented by gauge-invariant functionals of the spatial potentials only. The latter condition automatically ensures that the second constraint in (2) is satisfied, and that under any transformation of the type (4) a good functional \( \psi \) suffers the change

\[
\delta \psi = -\int d^2 x \left( \frac{\delta \psi}{\delta b^\alpha_\mu} \right) \nabla_\alpha \eta^\mu .
\]

Using Gauss' theorem and Leibnitz' rule for covariant differentiation one can write

\[
\delta \psi = -e^\mu \int d^2 x \frac{\delta \psi}{\delta b^\alpha_\mu} + i \int d^3 x (\nabla_\alpha \pi^\alpha \psi) \eta^\mu .
\]

By virtue of (2) \( \psi \) is gauge invariant provided \( \delta \psi/\delta b^\mu_\alpha \) is \( O(1/|x|^2) \) at spatial infinity, while the gauge invariance of \( \psi \) is sufficient to ensure that (2) is satisfied.

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(1) The variable \( x \) refers to the three-dimensional co-ordinates, \( x \) to the four-dimensional co-ordinate \( x^\mu \).