Geometro-Stochastic Quantization of Gauge Fields in Curved Space-Time (*)

E. Prugovečki
Department of Mathematics, University of Toronto - Toronto, Canada M5S 1A1

Summary. — It is shown that the geometro-stochastic method of quantization of massive fields in curved space-time can be extended to the massless cases of electromagnetic fields and general Yang-Mills fields. The Fock fibres of the massive case are replaced in the present context by fibres with indefinite inner products, such as Gupta-Bleuler fibres in the electromagnetic case. The quantum space-time form factor used in the massive case gives rise in the present case to quantum gauge frames whose elements are generalized coherent states corresponding to pseudounitary spin-one representations of direct products of the Poincaré group with the $U(1)$, $SU(N)$ or other internal gauge groups. Quantum connections are introduced on bundles of second-quantized frames, and the corresponding parallel transport is expressed in terms of path integrals for quantum frame propagators. In the Yang-Mills case, these path integrals make use of Faddeev-Popov quantum frames. It is shown, however, that in the present framework the ghost fields that give rise to these frames possess a geometric interpretation related to the presence of a super-gauge group that, in addition to the external Poincaré and Yang-Mills gauge degrees of freedom, involves also the internal ones related to choices of gauge bases within the quantum fibres.

PACS 11.10 - Field theory.
PACS 11.15.Tk - Other nonperturbative techniques.
PACS 11.30.Cp - Lorentz and Poincaré invariance.

(*) Supported in part by the NSERC Research Grant No. A5206.
1. – Introduction.

A previously developed \(^{(1)}\) method of geometro-stochastic quantization was recently applied \(^{(1)}\) to the quantization of massive fields in curved space-times \(M^4\). In the present paper we apply the same method to the quantization of those gauge fields in curved space-times \(^{(1)}\) which are of principal physical interest, namely to the electromagnetic and to the Yang-Mills fields related to \(SU(N)\) internal symmetries.

The main purpose of the geometro-stochastic method is to construct quantum geometries for quantum field theory in curved space-time. These geometries display the structure of soldered Hilbert bundles over base manifolds whose points represent the mean stochastic space-time locations for the origins of quantum frames, and whose parallel transport is executed by means of a quantum connection \(\nabla\) compatible with the Hermitian structure of that Hilbert bundle. Hence the method can be applied equally well in the nonrelativistic regime, except that, as shown in ref. \(^{(3)}\), in the latter case it is more convenient to adopt a five-dimensional Bargmann manifold \(M^5\) as base manifold for the quantum bundle. In the relativistic regime, the same approach leads to the possibility advocated by some authors \(^{(4)}\) of using the de Sitter rather than the Poincaré group as a gauge group, with the latter emerging from the spontaneously broken symmetry of the former achieved by requiring the invariance of a suitable Killing field \(^{(8)}\) under frame transformations. However, in the present paper in quantizing electromagnetic and Yang-Mills gauge fields we shall restrict ourselves to quantum frame bundles \(Q(M^4)\) over a four-dimensional pseudo-Riemannian base manifold \(M^4\) having all the properties \(^{(3)}\) of a general-relativistic space-time—in particular possessing a symmetric metric tensor \(g\), and therefore also a (unique) Levi-Civita connection \(\nabla\) compatible with it.

The geometro-stochastic method of field quantization contains three main stages \(^{(1)}\). The first stage involves the construction of a suitable vector space \(F\) that is to become the typical fibre of the would-be quantum bundle, and coincides with the procedure described in detail in ref. \(^{(1)}\), and labelled there «stochastic quantization» (not to be confused, however, with a couple of other methods of

---

\(^{(3)}\) E. PRUGOVEČKI: \textit{Class. Quantum Grav.}, 4, 1659 (1987).
\(^{(8)}\) H. P. KÜNZLE and C. DUVAL: \textit{Class. Quantum Grav.}, 3, 957 (1986).