The Gravitational Constant (*).

K. CAHILL

Department of Physics and Astronomy, University of New Mexico
Albuquerque, New Mexico 87131

(ricevuto il 21 Settembre 1983; manoscritto revisionato ricevuto il 3 Gennaio 1984)

PACS. 11.15 – Gauge field theories.

Summary. – Gravitational theories can be written in terms of rescaled fields without the Planck mass. The rescaled tetrads acquire the dimension of mass. The actual distribution of energy throughout space-time causes the tetrads to assume vacuum expected values of the order of the Planck mass, \( m_p \). Thus the gravitational constant, \( G = \frac{hc}{m_p^2} \), may be viewed not as a fundamental constant, but as a mass scale that is dynamically determined by the large-scale structure of the Universe.

How and why does Nature choose such vastly different mass scales? It is possible to use the Higgs mechanism to set arbitrary mass scales and to use supersymmetry to keep them fixed at their tree-level values. But this leaves open the question of why the mass scales have the values they do. For scalar fields have no reason to assume nonzero vacuum expected values. In fact the only fields that naturally do so are the tetrads.

Tetrad vacuum expected values, therefore, provide a natural way to establish mass scales and break symmetries \(^{(1,2)}\). It will be shown here that tetrads spontaneously break the general co-ordinate invariance and local Lorentz symmetry of gravitational theories by assuming vacuum expected values.

It will also be pointed out that gravitational theories can be written in terms of rescaled fields in such a way that the Planck mass never appears. The rescaled fields are dimensionless except for gauge fields and tetrads, both of which acquire the dimension of mass.

The v.e.v.'s of the tetrads are determined by the distribution of energy throughout space-time. It is, therefore, not possible to calculate tetrad v.e.v.'s without a knowledge of the actual energy distribution. However, to the extent that cosmological models plausibly yield Robertson-Walker metrics with \( g_{00} = 1 \), these same models yield v.e.v.'s of the rescaled tetrads of the order of the Planck mass \( m_p \). Thus


\(^{(2)}\) K. CAHILL: preprint DOE-UNM-831.1.

\(^{(1,2)}\) Supported in part by the U.S. Department of Energy under contract DE-AC04-81ER40042.
Newton's constant $G := \hbar c/m_\odot^2$ may be viewed not as a fundamental constant, but as a mass scale that is dynamically determined by the actual large-scale structure of the universe.

Theories of a Weyl spinor $\psi(x)$ interacting with a tetrad $e^\mu(x)$ have a two-fold symmetry. The action is invariant under a transformation $U$ that is both a local $SL_\infty$ transformation $T(x)$ of the field $\psi(x)$ and a general co-ordinate transformation of the space-time co-ordinates $x^\mu$. $U$ transforms the two-component Weyl spinor $\psi(x)$ and the tetrad $e^\mu(x)$ into

$$\psi(x)' = U^{-1} \psi(x) U = T(x) \psi(x')$$

and

$$e^\mu(x)' = U^{-1} e^\mu(x) U = \partial'_\nu x^\mu T^{-1\nu}(x) e^\nu(x') T^{-1}(x),$$

in which $\partial'_\nu = \partial/\partial x'^\nu$ and $x'$ is the image of $x$ under the associated general co-ordinate transformation. The action is invariant (1,2) even if there is no correlation whatsoever between the local Lorentz transformation $T(x)$ and the general co-ordinate transformation $x \rightarrow x'$.

Let us suppose that the tetrad $e^\mu(x)$ assumes an expected value in the $\psi$ vacuum state $|\Omega\rangle$ that represents the universe:

$$e^\mu(x)_c = \langle \Omega | e^\mu(x) | \Omega \rangle .$$

By eqs. (2)-(3) any transformation $U$ that leaves the state $|\Omega\rangle$ invariant must satisfy (2)

$$e^\mu(x)_c = \partial'_\nu x^\mu T^{-1\nu}(x) e^\nu(x') T^{-1}(x).$$

Its local Lorentz transformation $T(x)$ and its general co-ordinate transformation $x \rightarrow x'$ must be so synchronized as to leave the tetrad v.e.v.'s invariant.

The general co-ordinate transformation itself must be an isometry (2) of the space-time metric $g_{\mu\nu}(x)_c$,

$$g_{\mu\nu}(x)_c = \partial_\mu x'^\sigma \partial_\nu x'^\tau g_{\sigma\tau}(x')_c .$$

The isometries of the flat-space metric, $g_{\mu\nu}(x)_c = \eta_{\mu\nu}$, are just the global Poincaré transformations. So to the extent that empty space is flat over laboratory distances we may say that tetrad v.e.v.'s reduce the symmetry of the $\psi$ vacuum state $|\Omega\rangle$ to global Poincaré invariance.

The action for a Weyl spinor $\psi$ interacting with the gravitational field may be written as

$$S = \int d^4x (-g)^{1/2} \left[ (m_\odot/32\pi) \text{tr}(e^\mu F_{\mu\nu} h^\nu) + \frac{i}{2} \gamma^\dagger e^\mu D_\mu \psi + \text{h.c.} \right],$$

where $g$ is the determinant of $g_{\mu\nu}$, $h^\nu$ is the contragredient tetrad $h^\nu = \sigma_2 e^r x^2$, $D_\mu$ is the covariant derivative $D_\mu = \partial_\mu + A_\mu$ and $F_{\mu\nu}$ is the curvature tensor $F_{\mu\nu} = [D_\mu, D_\nu]$. The action is invariant when $U$ transforms $\psi$ and $e^\mu$ according to eqs. (1), (2), provided the connection $A_\mu$ transforms suitably (1).

Let us now rescale the fields by means of the following definitions:

$$m_{\mu\nu} = m_\odot^2 g_{\mu\nu},$$

$$\psi = m_\odot^{-1} \psi.$$