Solution of the Renormalization Group Equation and Ultraviolet Asymptotics of Gauge-Dependent Propagators in QCD.

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Summary. – The general solution of the RGE for gluon vertices in massless QCD is obtained for arbitrary \( \alpha \). It is shown that, contrary to the view prevailing in the literature, the ultraviolet behaviour of gluon vertices is discontinuous both in the gauge-fixing parameter \( \alpha \) and in the number of flavours.

We will consider the general \( n \)-point gluon vertex function

\[
I^{(n)}(p, g, \alpha, \mu), \quad p = (p_1, \ldots, p_n),
\]

where \( g \) and \( \alpha \) are, respectively, the coupling constant and the gauge-fixing parameter, both renormalized at the scale \( \mu \), and all momenta are in the Euclidean region.

It is convenient to work with a dimensionless object

\[
\tilde{I}^{(n)}(p) = (P_1^2)^{-2} \prod_{i=1}^{n} (P_i^2 + (\alpha/\mu)^2) I^{(n)}(p).
\]

Scaling the four-momenta in the standard way we get the equality

\[
\tilde{I}^{(n)}(\exp[t] p, g, \alpha, \mu) = \tilde{I}^{(n)}(p, g, \alpha, \exp[-t] \mu),
\]

the differential RG statement of which is

\[
\left[ -\frac{\partial}{\partial t} + \beta \frac{\partial}{\partial g} + \delta \frac{\partial}{\partial \alpha} - n \gamma \right] I^{(n)} = 0
\]
with the conventional definitions of the renormalization group functions $\beta$, $\delta$ and the anomalous dimension $\gamma_{\nu}$ (1).

Gauge invariance implies that

$$\alpha = Z_{3}^{-1} \alpha_{B}, \quad \delta = -2x\gamma_{\nu},$$

where the index $B$ denotes unrenormalized quantities. Equation (4) can be easily solved by the method of characteristics (see e.g. (2)).

The characteristics equations are

$$-\frac{dt}{\beta(g)} = \frac{dx}{\delta(g, \alpha)} = \frac{df^{(n)}}{n\gamma_{\nu}(g, \alpha) f^{(n)}}$$

and can be simultaneously integrated to give the characteristic surfaces (the system (6) is equivalent to the partial differential equation (4))

$$t + \int \frac{dx}{\beta(x)} = A,$$

$$f(g, \alpha) = B,$$

$$f^{(n)}x^{n/2} = c,$$

where $A, B, C$ are arbitrary integration constants and $f$ is a parametric solution of the equation

$$\frac{dg}{\beta(g)} = -\frac{dx}{2x\gamma_{\nu}(g, \alpha)}.$$

(We choose to work in the MS scheme, so that $\beta$ is independent of $x$ (3)).

In paper (4) West considers a special case of (4) with $n = 2$ and $x = 0$ (so that the $x$-derivative is absent from the RGE). However, the characteristic equations obtained in (4) contain a sign error.

Returning to the more general case $x \neq 0$, the general solution of (4) is

$$f^{(n)}(\exp[t] p, g, \alpha, \mu) = x^{-n/2} F \left( z + \int \frac{dx}{\beta(x)}, f(g, \alpha, \mu) \right),$$

where $F$ is an arbitrary (smooth) function. We get rid of the unknown function $F$ by