Recursive Largest Claims Reinsurance Rating, Revisited

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1. Introduction

During the past eleven years the author developed a new mathematical theory for a class of certain generalized reinsurance treaties, the so-called generalized largest claims reinsurance treaties (see e.g. Kremer (1984), (1985), (1986a), (1988a), (1990a), (1990b), (1992)). One part of that theory consists of a general premium rating theory that separates into direct, bounding and recursive techniques (for a survey see Kremer (1988b)). The recursive rating procedure in Kremer (1986a) is already fairly general but in its approximate form (3.3) (see page 105 in Kremer (1986a)) still crude. One would like to have a handy modification of that recursion giving improved results in typical situations. Checking carefully the paper Kremer (1986a) again, one can find out that the results of part 3 can not be applied to one well-know special treaty, the so-called ECOMOR cover. Consequently one likes to extend the results of that article such that also for the ECOMOR treaty a handy recursive rating procedure can be deduced. In the following additional investigations an adequate modification of the previous handy approximate recursive rating method is given and also an adequate extension that can be specialised to the ECOMOR cover. Tables with numerical results are given for the classical largest claims and the ECOMOR treaty, demonstrating the utility of the derived additional results.

2. The reinsurance cover

Consider a collective of risks and let \(N\) denote the random variable of the number of claims. The corresponding claims amounts are described by the random variables \(X_1, X_2, X_3, \ldots\). Suppose all random variables are defined on one and the same probability space \((\Omega, \mathcal{A}, P)\). Denote by:

\[
X_{N:1} \leq \ldots \leq X_{N:N}
\]

the claims ordered in increasing size. Furthermore let \(c_1, c_2, c_3, \ldots\) be real constants and the random variable:

\[
R_N = \sum_{i=1}^{N} c_i \cdot X_{N:N-i+1},
\]

describing a certain claims amount to be paid by the reinsurer. Consequently the family \((c_1, c_2, \ldots)\) defines a reinsurance treaty, called generalized largest claims cover. One can take a natural number \(p\) and gets with the special family

\((c_1, c_2, c_3, \ldots, c_p, 0, 0, \ldots, 0, \ldots)\)

a more special treaty that one can call (generalized) largest claims cover with parameter \(p\) (in short GLC\((p)\)). For the more special choice:

\[
c_i = 1, \quad \text{for all } i,
\]

one gets the classical largest claims treaty (in short LC\((p)\)), covering the \(p\) largest claims (see e.g. Ammeter (1964) or Kremer (1988b)).
Suppose like in Kremer (1986a) in the sequel that

1) the claims number $N$ satisfies with given parameters $a$, $b$ the recursion:

$$P(N = n) = P(N = n - 1) \cdot (a + b/n)$$

for $n = 1, 2, 3, \ldots$

2) the claims sizes $X_1, X_2, X_3, \ldots$ are identically distributed with continuous distribution function $F$.

3) the random variables $N, X_1, X_2, X_3, \ldots$ are stochastically independent.

Under these additional conditions one investigates the net premium $m_p$ of the GLC(p) treaty:

$$m_p = \sum_{i=1}^{p} c_i \cdot E(X_{N:i+1})$$  \hspace{1cm} (2.1)

(with the convention $X_{N:i+1} = 0$, if $N < i$).

3. A modified recursive rating

It was proved in Kremer (1986a) that one has the recursion:

$$m_p = m_{p-1} \cdot \left(1 + K_p \cdot \left(1 + \left(\frac{1 - a}{p - 1}\right) \cdot \lambda\right)\right) - m_{p-2} \cdot \left(K_p \cdot \left(\frac{1 - a}{p - 1}\right) \cdot \lambda\right)$$

$$- E(N \cdot X_{N:n-p+2}) \cdot \left(\frac{1 - a}{p - 1}\right) \cdot c_p,$$

with start:

$$m_1 = c_1 \cdot E(X_{N:n}),$$

$$m_2 = c_2 \cdot E(X_{N:n-1}) + m_1,$$

and the conventions:

$$K_p = c_p / c_{p-1},$$

$$\lambda = E(N) = \frac{a + b}{1 - a}.$$

In that paper the quantity $E(N \cdot X_{N:n-p+2})$ was asymptotically equivalently replaced by:

$$\lambda \cdot E(X_{N:n-p+2}) = \lambda \cdot (m_{p-1} - m_{p-2}) / c_{p-1},$$

implying the very handy approximate recursion:

$$m_p \approx m_{p-1} \cdot (1 + K_p) - m_{p-2} \cdot K_p,$$  \hspace{1cm} (3.1)

It was told that one has:

$$E(N \cdot X_{N:n-p+2}) \geq \lambda \cdot E(X_{N:n-p+2}),$$  \hspace{1cm} (3.2)

meaning that the right hand side in (3.1) gives an upper bound for $m_p$ in case that:

$$c_p > 0$$  \hspace{1cm} (3.3)

and a lower bound in case that:

$$c_p < 0.$$