A Polynomial Approach to Gravitational Theories.

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Summary. — We emphasize that, in a classical theory, invariant under general co-ordinate transformations, the nonpolynomial dependence can be reabsorbed by a suitable fields rescaling, if the theory is scale invariant. The new fields show, in the dilatation limit, the canonical UV dimensions. We derive the BRS differential operator of the new theory and we show here the results we have obtained in its quantum extensions. The proofs will be given in forthcoming more technical papers.

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The renormalization of gravitational theories is one of the most actual problems in theoretical physics and lives today a renewal, due to the great progress in gauge quantum field theory.

In fact the results obtained in axiomatic renormalization theory, together with the advantages achieved by the differential geometrical methods, lead to new insights in gauge field dynamics.

Now, if we suppose that gravitational theories admit a gauge symmetry group, which, in its most general extension, is generated by the invariance under co-ordinate reparametrization, we can hope to exploit the experience we got from gauge theories.

Several trials have preceded ours (1), and we are confident that in the following years someone will reach the goal.

In this paper we propose an alternative point of view to set the gravitational theory, borrowing an idea which is due to Weyl and used in weak gravitational limit in ref. (2).

The starting point arise from the analysis that, in the dilatation invariance limit, the usual Einstein transformations induce different behaviours from the ones in the flat-space limit.

If we make a rescaling of the fields in order to get the usual UV dimensions, we find that the nonpolynomial dependence in the gravitational action drops out.

At this stage the theory becomes invariant under a new symmetry, which is induced by mixed Einstein-conformal transformation.

It is then a standard routine to build the BRS operator of the theory, but the most important result in this paper is that a deep link relates the tensorial content of the objects of the theory, to its UV dimensions, Weyl weight ad its Faddeev-Popov charge.

This constraint fixed the dimensionality of our polynomial Lagrangian as equal to four.

In forthcoming papers we shall study the quantum extension of our model with the aid of cohomological techniques.

1. - Classical theory.

Gravity is considered today as the gauge theory which is invariant, in its simplest version, under the local co-ordinates reparametrisation $x^\mu \rightarrow x'^\mu = x^\mu - \lambda^\mu(x)$. This suggests a deep similarity in the approach and the methods with non-Abelian gauge theories.

The first difficulty comes, in this program, from the fact that the gravitational actions are not, as well known, polynomial functionals, so many theorems of the ordinary Lagrangian quantum field theories cannot be applied.

This is explicitly shown in simple examples of gravitational classical actions (in a four-dimension space-time) which will be useful in the following:

\begin{align}
(1.1a) \quad & I_{\text{bosons}}^{(1)} = \int d^4x \sigma(x) \tilde{g}^{\mu\nu}(x) \partial_\mu \tilde{\phi}(x) \partial_\nu \tilde{\phi}(x), \\
(1.1b) \quad & I_{\text{fermions}}^{(1)} = \int d^4x \sigma(x) \tilde{\Psi}(x) \gamma^a \tilde{\gamma}^\mu_a(x)(\partial_\mu + \omega^\mu_\alpha(x) \Sigma^\alpha)(\tilde{\Psi}(x), \\
(1.1c) \quad & I_{\text{vectors}}^{(1)} = \int d^4x \sigma(x) \tilde{g}^{\mu\nu}(x) \tilde{\gamma}^a(x) F_{\mu\nu}(x) F^a_{\nu\sigma}(x),
\end{align}

where

\[ \sigma(x) = \sqrt{-\det |\tilde{g}_{\mu\nu}(x)|} \quad \text{and} \quad \tilde{g}^{\mu\nu}(x) = \tilde{\gamma}^a(x) \tilde{\gamma}_a(x), \]