On the Unification of Matter Fields.

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Summary. — Heisenberg-type nonlinear unifying equations in difference form are derived from a well-known basis. It is also noticed that electromagnetic self-interaction can give rise to other forms of self-interaction.

Heisenberg and his co-workers (1-3) have tried to explain elementary particles on the basis of a unifying nonlinear spinor equation. But the wave equation they proposed, is more from an intuitive basis than from any deductive sequence. In this note we shall derive one Heisenberg-type of nonlinear wave equation from a well-known basis.

As for notation Roman indices will run from 1 to 4. Spinor indices will be suppressed. Summation convention will be followed. Metric tensor is taken to be $\eta_{ab} = \text{diag} (-1, -1, -1, 1)$. An event will be denoted in short by $x$. Partial derivatives are written as $\partial_a \psi \equiv \partial \psi / \partial x^a$. Units are so chosen that $\hbar = c = 1$.

We shall start from the combined electron-positron-electromagnetic field equations

$$\begin{align*}
&\left[ -i\gamma^a (\partial_a + ieA_a) + mI \right] \psi = 0 , \\
&\bar{\psi} [i\gamma^a (\partial_a - ieA_a) + mI] = 0 , \\
&\partial_a \partial^a A_b = e\bar{\psi} \gamma_b \psi , \\
&A_b (x) = e \int D_\gamma (x - x') \bar{\psi} (x') \gamma_b \psi (x') \, dx' ,
\end{align*}$$

(1)

where $\psi$ is the electron-positron field, $A_\alpha$ is the electromagnetic field, $\gamma^\alpha$s denote Dirac matrices, $\bar{\psi} = \psi^\dagger \gamma^4$, and $D_\mu$ is the Feynman Kernel function. We notice that from the set of eqs. (1) electromagnetic fields $A_\alpha$ can be eliminated and the resulting nonlinear $\psi$-equation in the lim $m \to 0$ becomes

$$-i\gamma^\alpha \bar{\psi} \gamma_\alpha \psi(x) + e^2 \left[ \int D_\mu(x - x') \bar{\psi}(x') \gamma_\alpha \psi(x') \, d^4x' \right] \gamma^\alpha \psi(x) = 0.$$  

Equation (2) will form the basis of our unified field equation.

It may be mentioned that eq. (2) can be enlarged to include other interactions by adding terms like

$$\sum \epsilon(\alpha) \left[ \int D_\mu \bar{\psi} \Gamma(\alpha) \psi \, d^4x' \right] \Gamma^{\alpha\beta} \psi.$$  

where $\Gamma(\alpha)$ are Eddington matrices. Linearized (Birkhoff-type) gravitation may be incorporated by adding to (2) a term

$$\frac{G^2}{8} \left[ \int D_\mu \bar{\psi} \gamma_\alpha \gamma_\beta \psi + \bar{\psi} \gamma_\alpha \gamma_\beta \psi - \bar{\psi} \gamma_\beta \psi - \bar{\psi} \gamma_\beta \psi \right] \left[ \bar{\psi} \gamma_\alpha \psi + \gamma_\beta \bar{\psi} \gamma_\beta \psi \right].$$  

Now we shall transfer the eq. (2) from the background of continuum to the cellular space-time. We shall employ the absolute units, i.e., $\hbar = c = 1$, so that all physical quantities are expressed as pure numbers. In cellular space-time each co-ordinate $x^\alpha = 0, \pm 1, \pm 2, \ldots$, and various difference operations may be defined as

$$\Delta = f(...x + 1...) - f(...x...),$$  

$$\Delta' = f(...x...) - f(...x - 1...),$$  

$$\Delta'' = \frac{1}{2}[f(...x + 1...) - f(...x - 1...)].$$  

The analogue of (2) in the cellular space-time is (cf. Das (4))

$$-i\gamma^\alpha \bar{\psi} \gamma_\alpha \psi + e^2 D_\mu(\bar{\psi} \gamma_\alpha \psi) \gamma^\alpha \psi = -e^2 \left[ \sum \frac{D_\mu(x, x') \bar{\psi}(x') \gamma_\alpha \psi(x')}{x', x} \right] \gamma^\alpha \psi(x),$$

$$D_\mu(x, x') = \frac{1}{(2\pi)^4} \int \int \int \int \frac{\exp [ik_\mu(x^\mu - x'^\mu)]}{-4\pi \sin (k_b/2) \sin (k/2)},$$

$$\Delta \Delta' D_\mu(x, x') = \delta^4_{xx'},$$

$$D_\mu = D_\mu(x, x), \quad |D_\mu| < \infty.$$  

(4) A. Das: Nuovo Cimento, 18, 482 (1960).