A Classical Solution.

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Summary. — A linear potential is found to be a static «electric» solution to the classical Yang-Mills gauge theory.

Classical solutions to the equation of motion of the Yang-Mills gauge theory

\[ f_{\mu \nu \rho} + gb_{\rho} \times f_{\mu \nu} = - j_{\nu} \]

have been studied extensively in recent years (1,2). Most of them deal with the sourceless \( j_{\nu} = 0 \) case (1), and some of them deal with the case in which the vector field \( b_{\nu} \) couples with some scalar meson field (2).

We wish to use eq. (1) to study the nonrelativistic quark model in which the meson is considered as a qq bound state, and the colour gluons provide the binding of the quarks. The gluon field is then generated by a static quark source

\[ \sigma_{\alpha} = - i j_{\alpha} = 4\pi g g^{\alpha}(x - x_{0})\eta_{\alpha}, \quad j_{l} = 0, \quad l = 1, 2, 3, \]

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where $\alpha = 1, \ldots, 8$ is the $SU_3$ index, and $\eta_{\alpha}^a$ is the colour charge of the quark $q_a$. The Latin letters at the beginning of the alphabet ($a, b, c, \ldots$) are used for flavour, and letters in the middle ($i, j, k, l, \ldots$) are for colour matrix elements and space indices. It is found that a static «electric» solution to the time component of eq. (1) is simply

$$\begin{cases} b_{\alpha} = g r \eta_{\alpha}^a + c_1 T_{\alpha} \cdot e_2 , \\ b_i^\alpha = 0 , \end{cases}$$

where $T^{\alpha} = \varepsilon_{ijk} \lambda_{ij}^\alpha \sigma_z$. Explicitly the $T$-matrix has the following form:

$$T_1 = \begin{pmatrix} 0 & x_2 & 0 \\ -x_2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$T_2 = \begin{pmatrix} 0 & -ix_3 & 0 \\ -ix_3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$T_3 = T_8 = 0 ,$$

$$T_4 = \begin{pmatrix} 0 & 0 & -x_2 \\ 0 & 0 & 0 \\ x_2 & 0 & 0 \end{pmatrix},$$

$$T_5 = \begin{pmatrix} 0 & 0 & ix_2 \\ 0 & 0 & 0 \\ ix_2 & 0 & 0 \end{pmatrix},$$

$$T_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & x_1 \\ 0 & -x_1 & 0 \end{pmatrix},$$

$$T_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -ix_1 \\ 0 & -ix_1 & 0 \end{pmatrix}.$$