A Rigorous Bound for the $e^+e^- \rightarrow \pi^0\gamma$ Cross-Section (*)

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Summary. — Using only the analyticity of the $\gamma\gamma\pi^0$ form factor and the experimental lifetime of the $\pi^0$, we determine the bound

$$\int_{4m^2}^{\infty} ds \sigma_{tot}(s) > 35.3 \mu b (\text{MeV})^4 (4m^2) k^2 (5.91 + k^2 + 4.44k + 256a^2 - 32ak - 70.9a)$$

where $\sigma_{tot}(s)$ is the total cross-section for $e^+e^- \rightarrow \pi^0\gamma$, $k$ is an arbitrary real number, and $a$ is the slope of the form factor at $s = 0$. Bounds are also obtained for the isoscalar and isovector cross-sections separately.

By means of only analyticity of various form factors and positivity, many bounds on physically interesting quantities have recently been derived. A bound involving derivatives of $k_{13}$ form factors has been obtained by Li and Pagels (1). They (2) subsequently derived a bound on an undifferentiated $k_{13}$ form factor using a technique originally due to Meiman (3). Okubo (4,5) has presented an elegant generalization of their method which leads to improved bounds on the $k_{13}$ parameters. In addition, bounds have been obtained on the electromagnetic radius of the pion (6,7), the hadronic contributions to the anomalous magnetic

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(5) S. Okubo: to be published in Phys. Rev. D.

(6) D. N. Levin and V S. Mathur: private communication.
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moment of the muon (\textsuperscript{7}) and the dimensions of scale breaking interactions (\textsuperscript{8}). In this note we consider the $\gamma\gamma\pi^0$ form factor in order to derive bounds on an integral over the cross-section for $e^+e^-\rightarrow \pi^0\gamma$. We assume that this process proceeds via the exchange of a single, virtual photon. With regard to this assumption we note that the two-photon mechanism discussed by Brodsky, Kinoshita and Terazawa (\textsuperscript{9}), in which the incident leptons emit two virtual photons which in turn annihilate to produce the nonleptonic final state, should not be significant for this process because the $\gamma\gamma\rightarrow \pi^0\gamma$ amplitude is zero by charge-conjugation invariance.

If the $\gamma\gamma\pi^0$ form factor $f(s)$ is defined by the relation

\begin{equation}
(4p_\gamma p'_\gamma V V') \langle \gamma | f(0) | \pi^0(p), \gamma(p') \rangle = \frac{f(p + p')}{m_{\pi}} e_{\mu\nu} p^\mu p'^\nu e^\gamma,
\end{equation}

where $e^\gamma$ is the photon polarization vector and $e_{\mu\nu}$ is the completely antisymmetric Ricci tensor, and the one-photon exchange approximation is valid for the reaction $e^+e^-\rightarrow \pi^0\gamma$, then one has the expression (\textsuperscript{10})

\begin{equation}
\sigma_{tot}(s) = \frac{1}{6} \pi \alpha^2 \frac{(s - m_{\pi}^2)^3}{s^2 m_{\pi}^2} |f(s)|^2
\end{equation}

for the total cross-section. Here $s$ is the square of the center-of-mass energy. After multiplying both sides of eq. (2) by $s^k$ and integrating from $s = 4m_{\pi}^2$ to $\infty$, we obtain

\begin{equation}
\int_{4m_{\pi}^2}^{\infty} ds s^k \sigma_{tot}(s) = \frac{\pi \alpha^4}{6 m_{\pi}^2} \int_{4m_{\pi}^2}^{\infty} ds s^{k-1}(s - m_{\pi}^2)^3 |f(s)|^2.
\end{equation}

The real number $k$ is arbitrary to the extent the integrals involved exist.

The form factor $f(s)$ is an analytic function of $s$ in the complex $s$-plane with a branch cut from $s = 4m_{\pi}^2$ to $\infty$. At $s = 0$, in the unphysical region for $e^+e^-\rightarrow \pi^0\gamma$, the form factor describes the two-photon decay of the $\pi^0$:

\begin{equation}
\frac{1}{\tau} = \frac{\alpha}{16 m_{\pi}} |f(0)|^2,
\end{equation}

where $\tau$ is the mean life.

\textsuperscript{(7)} D. R. Palmer: to be published in Phys. Rev. D.

