Flow Transition through Several Shock Surfaces.

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Summary. — Landau and Lifshitz (1) have mentioned the following result. If two (or more) successive shock waves take a gas from state 1 to state 2 and from there to state 3, the transition from state 1 to state 3 cannot, in general, be effected by the passage of any one shock wave. Mishra (2) has shown that in order that the transition from state 1 to state 3 be effected by the passage of a single shock, the necessary and sufficient condition is that the three shock surfaces be tangential to one another. The object, here, is to find conditions under which the flow transition through \((m - 1)\) shocks in a conducting gas may be possible.

1. Jump conditions.

The jump conditions for the flow of a conducting gas are given by

\begin{align}
Q_{i1} u_{1n} & = Q_{i2} u_{2n}, \\
H_{1n} & = H_{2n}, \\
Q_{i1} u_{1n} [u_i] & = - [p^* u_i] + \frac{1}{4\pi} H_{1n} [H_i], \\
[H_i u_n] & = H_{1n} [u_i], \\
Q_{i1} u_{1n} \left[ \frac{u^2}{2} + h + \frac{H^2}{4\pi\epsilon} \right] - [H_i u_i] H_{1n} & = 0,
\end{align}

where $\rho$, $u_1$, $u_2$, $H_1$, $H_2$, $p^*$ and $h$ are respectively the density, velocity components of a fluid particle, normal fluid velocity, components of the magnetic field, normal component of the magnetic field, total pressure and enthalpy. Also, $n_i$ are the unit components normal to the shock and $\lbrack f \rbrack = f_2 - f_1$.

2. - Discussion of the problem.

Consider $(m-1)$ shocks such that the regions between the consecutive shocks be infinitely small. Let us denote the region in front of the first shock by 1, between the first and second by 2, between the $(m-2)$-th and the $(m-1)$-th by $(m-1)/$ and behind the $(m-1)$-th by $m/$. As a result of (1.1)-(1.5), jump conditions for regions $\alpha/$ and $(\alpha+1)/$ are given by

\begin{align}
(2.1) & \quad q_{\alpha}/u_{\alpha n} = q_{\alpha+1/}u_{\alpha+1 n}, \\
(2.2) & \quad H_{\alpha n} = H_{\alpha+1 n}, \\
(2.3) & \quad q_{\alpha}/u_{\alpha n}(u_{\alpha+1/} - u_{\alpha/}) = -(p^*_{\alpha+1/} - p^*_{\alpha/})u_{\alpha/} + \frac{H_{\alpha n}^2}{4\pi} (H_{\alpha+1/} - H_{\alpha/}), \\
(2.4) & \quad H_{\alpha+1/}u_{\alpha+1 n} - H_{\alpha/}u_{\alpha n} = H_{\alpha n}(u_{\alpha+1/} - u_{\alpha/}), \\
(2.5) & \quad q_{\alpha}/u_{\alpha n}\left(\frac{u_{\alpha+1/}^2}{2} + h_{\alpha+1/} + \frac{H_{\alpha+1/}^2}{4\pi q_{\alpha+1/}} - \frac{u_{\alpha/}^2}{2} - h_{\alpha/} - \frac{H_{\alpha/}^2}{4\pi q_{\alpha/}}\right)
- H_{\alpha n}(H_{\alpha+1/}u_{\alpha+1/} - H_{\alpha/}u_{\alpha/}) = 0.
\end{align}

If there is a flow transition from the region $1/$ to $m/$, then similar to jump conditions for adjoining regions, we have

\begin{align}
(2.6) & \quad q_{1/}u_{1/} \cos \theta_{1/} = q_{m/}u_{m/} \cos \varphi_{m/}, \\
(2.7) & \quad H_{1/} \cos \theta_{1/} = H_{m/} \cos \varphi_{m/}, \\
(2.8) & \quad q_{1/}u_{1/} \cos \theta_{1/}(u_{m/} - u_{1/}) = -(p^*_{m/} - p^*_{1/})u_{1/} + \frac{1}{4\pi} H_{1/} \cos \theta_{1/}(H_{m/} - H_{1/}), \\
(2.9) & \quad H_{m/}u_{m/} \cos \varphi_{m/} - H_{1/}u_{1/} \cos \theta_{1/} = H_{1/} \cos \theta_{1/}(u_{m/} - u_{1/}), \\
(2.10) & \quad q_{1/}u_{1/} \cos \theta_{1/}\left(\frac{u_{m/}^2}{2} + h_{m/} + \frac{H_{m/}^2}{4\pi q_{m/}} - \frac{u_{1/}^2}{2} - h_{1/} - \frac{H_{1/}^2}{4\pi q_{1/}}\right)
- H_{1/} \cos \theta_{1/}(H_{m/}u_{m/} - H_{1/}u_{1/}) = 0.
\end{align}