Exact Determination of a Phenomenological Separable Interaction (*)

M. Gourdin and A. Martin
Laboratoire de Physique, Ecole Normale Supérieure - Paris

Summary. — The method of Muskhelishvili applied by Omnes to the problem of one meson production by mesons is used to solve the integral equation determining a phenomenological separable interaction from the energy dependence of an experimental phase shift in a collision problem. The interaction in momentum space is given by an expression containing only one integral. The results obtained in a previous paper concerning the existence and multiplicity of the solutions are confirmed. The method is checked by an explicit calculation in the case when the phase shift is exactly given by the shape independent approximation.

1. — Introduction.

In a previous paper (1), we have shown that the determination of a phenomenological separable interaction fitting the energy dependence of a phase shift \( \delta \) obtained from the experimental study of a collision problem could be reduced to the problem of finding the solutions of the following integral equation:

\[
(1) \quad f(k) = g(k) \left[ 1 + \int_{\delta}^{\infty} \frac{f(p)}{p^2 - k^2} dp \right],
\]

(*) Supported in part by the United States Air Force through the European Office Air Research and Development Command.

(1) M. Gourdin and A. Martin: Nuovo Cimento, 6, 757 (1957). This paper will be designated hereafter as A.
where, when we restrict ourselves to $S$ states:

$$
\begin{align*}
    f(k) &= -\frac{2k}{\pi} \tan \delta, \\
    i(k) &= \varepsilon \frac{M}{2\pi^2} k^2 v^2(k),
\end{align*}
$$

(2)

$\varepsilon$ is the sign of the non-local separable interaction; $v(k)$ is the Fourier transform of the interaction.

We were able to determine solutions of this equation when $g(k)$ was any rational function of the energy $k^2$. Recently, however, Omnès (2) has proposed a very elegant method to solve an integral equation which has strong similarity with equation (1):

$$
\varphi(x) = \lambda(x) + \frac{1}{\pi} \int_{-\infty}^{\infty} h^*(y) \varphi(y) \, dy,
$$

(3)

where $h(x) = \exp [i \delta(x)] \sin \delta(x)$.

It can easily be seen that equation (1) can be brought into the shape (3), provided one makes the change of variable:

$$
x = k^2 + 1
$$

and the change of function:

$$
\varphi(x) = \frac{f(x)}{g(x)} \left[ 1 + i\pi \frac{g(x)}{2\sqrt{x} - 1} \right].
$$

Equation (1) then becomes:

$$
\varphi(x) = 1 + \frac{1}{\pi} \int_{-\infty}^{\infty} h_0^*(y) \varphi(y) \, dy,
$$

(4)

where

$$
h_0(x) = \frac{\pi g(x)}{2\sqrt{x} - 1} = -\sin \delta \exp [i\delta].
$$