Mass Renormalization for a Vector Meson.

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Summary. — We present our results on the mass renormalization of a vector meson within the framework of shadow-state theory based on the techniques of the Feynman diagram, the unitarity and dispersion relations. We also contrast the analytic result obtained from the conventional renormalization prescription and the piecewise analytic result from the shadow-state theory.

1. - Introduction.

Presently there are two distinct approaches to the problem of resonances and their decay. The first is a more conventional one, where one explicitly accounts for interactions among these particles which are responsible for the formation and the decay of the resonances. Recently, a different approach has been suggested, in which an unstable particle is treated as an "isolated" particle represented by a state in the nonunitary irreducible representation of the Poincaré group associated with a complex mass \(^1\). The latter, although it has mathematical elegance, from a physical point of view we believe to be unsatisfactory. For example, in this approach the dynamical origin of the resonance and its decay are completely ignored, consequently much of the physical insights provided by these phenomena, which are responsible for the very complexity of the mass, are lost. In our present work, we adhere to the first approach. We consider the mass renormalization of a vector meson.

In quantum field theory the «prescription» for the mass renormalization is well known. To calculate the self-energy correction to a bare propagator, we recall that one takes a chain approximation summing over the iterative series

\[ \cdots + \quad + \quad + \]

Fig. 1. – A bubble-diagram series.

of the self-energy diagrams—the bubble diagrams, see Fig. 1. The final expression for the square of the «dressed mass» for a boson propagator has the form

\[ m^2 \equiv m^2 + \sum \Sigma(p^2) \quad \text{with} \quad m^2 = m^2 + \text{Re} \Sigma(p^2), \]

where \( \Sigma(p^2) \) is the bubble-diagram contribution. On the right-hand side, \( m \) is the renormalized mass or the physical mass, while \( 2i \text{Im} \Sigma(p^2) \) is the discontinuity across the branch cut of the two-body intermediate state in the bubble diagram.

For a local interaction, in general the bubble diagram diverges. For example, for spin-0 propagator, with the spinless intermediate particles, the Feynman integral diverges logarithmically. If we denote the cut-off in the Feynman integral by \( M \), and \( \Sigma(p^2) = \lim_{M \to \infty} \Sigma(p^2, M) \), then for the \( S \)-wave propagator as \( M \to \infty \)

\[ \Sigma^S(p^2, M) \to \log M, \]

while for a spin-one propagator

\[ \Sigma^P(p^2, M) \to M^2. \]

In the conventional renormalization prescription, one assumes that the bare mass \( m_0 \) is infinite; compensates the infinity in \( \text{Re} \Sigma(p^2) = \text{Re} \Sigma(p^2, \infty) \) with the sum \( m^2 + \text{Re} \Sigma(p^2, \infty) \) giving the observed mass squared \( m^2 \).

In the past few years, there has been renewed interest in this mass renormalization problem. Various authors have suggested dynamical models to get rid of the ugly infinity of the bare mass. This can be achieved for instance by the introduction of the shadow fields and indefinite metric into the theory \( ^2 \).