Example of a Macroscopical Classical Situation that Violates Bell Inequalities.

D. Aerts

Theoretische Natuurkunde Vrije Universiteit Brussel - Pleinlaan 2, 1050 Brussel, Belgium

(ricevuto il 5 Aprile 1982)

Summary – We give an example of a classical macroscopical situation that violates Bell inequalities. The example shows a certain analogy with the system composed of two spin-\(\frac{1}{2}\) particles in the singlet spin state.

It was Bell who emphasized that if one wants to make a hidden-variable formalism that reproduces the results predicted by quantum mechanics for two spin-\(\frac{1}{2}\) particles in the singlet spin state, then this hidden-variable formalism must be nonlocal (\(^1\)). This study of Bell was done on the hand of a certain type of inequality which is now called the Bell inequality. It is often thought that this Bell inequality is only violated by systems described by quantum mechanics. We give an example of a classical macroscopical situation that violates Bell inequalities. The example shows a certain analogy with the system composed of two spin-\(\frac{1}{2}\) particles in the singlet spin state. This example has been found as a result of a study of separated physical systems (\(^2\) and \(^3\)) and supports what is shown in (\(^2\)) and (\(^3\)), namely that Bell inequalities are satisfied if and only if the two systems under consideration are separated. Whether the systems are quantal or classical is of no importance.

Before giving the example, let us state the definitions necessary to formulate Bell inequalities.

We have a physical system described by a hidden-variable formalism (in the terminology of (\(^2\)) and (\(^3\)), the system is classical). Let us call \(\Sigma\) the set of states of the system. We will denote yes-no experiments by the greek letters \(\alpha, \beta, \gamma, \delta\). To every yes-no experiment \(\alpha\) we make correspond a random variable \(X_\alpha\).

\[
X_\alpha : \Sigma \to \{-1, +1\}
\]

such that

\[
X_\alpha(p) = +1, \quad \text{if we have the answer "yes"},
\]

\[
X_\alpha(p) = -1, \quad \text{if we have the answer "no"}.
\]

\(^1\) J. S. Bell: Physics, 1, 195 (1965).


If we have two yes-no experiments \( \alpha \) and \( \beta \), it can happen that the experimental arrangements used to perform \( \alpha \) and \( \beta \) are not incompatible, in this sense it is possible to perform a coincidence experiment on the system. What we do is to perform the experiment \( \alpha \) and \( \beta \) in such a way that the answers of both experiments are obtained at the same time. We define then

\[
X_{\alpha \beta}(p) = +1, \quad \text{if we get the answer \{yes, yes\} or \{no, no\}}
\]

for the coincidence experiment,

\[
X_{\alpha \beta}(p) = -1, \quad \text{if we get the answer \{yes, no\} or \{no, yes\}}
\]

for the coincidence experiment.

In general there will be no connection between \( X_{\alpha \beta}(p) \) and \( X_{\alpha \delta}(p) \) and \( X_{\delta \beta}(p) \) since the coincidence experiment of \( \alpha \) and \( \beta \) is a new experiment.

We suppose now that we have four yes-no experiments, \( \alpha, \beta, \delta \) and \( \gamma \), such that it is possible to perform coincidence experiments of \( \alpha \) and \( \beta \), of \( \alpha \) and \( \gamma \), of \( \delta \) and \( \beta \) and of \( \delta \) and \( \gamma \). Bell considers the situation where \( \alpha \) and \( \delta \) are experiments done on a system \( S_1 \) and \( \beta \) and \( \gamma \) experiments done on a system \( S_2 \) such that \( S_1 \) and \( S_2 \) are localized in different regions of space and such that

\[
\begin{align*}
(1) & \quad X_{\alpha \beta}(p) = X_{\alpha}(p)X_{\beta}(p), \\
(2) & \quad X_{\alpha \gamma}(p) = X_{\alpha}(p)X_{\gamma}(p), \\
(3) & \quad X_{\gamma \beta}(p) = X_{\gamma}(p)X_{\beta}(p), \\
(4) & \quad X_{\delta \gamma}(p) = X_{\delta}(p)X_{\gamma}(p).
\end{align*}
\]

He derives then an equality of the following type:

\[
(5) \quad |X_{\alpha \beta}(p) - X_{\alpha \gamma}(p)| + |X_{\delta \beta}(p) + X_{\delta \gamma}(p)| < 2.
\]

What we show in (5) is indeed that requirements (1)-(4) imply that the systems \( S_1 \) and \( S_2 \) are separated. They are not necessarily satisfied for nonseparated systems.

Let us now give the example of a macroscopical situation that violates Bell inequalities. Let us first define the physical system and the yes-no experiments that we want to consider. We consider a physical system that is a vessel that contains water. We consider the following experiments.

**experiment \( \alpha \):** it consists in testing whether the volume of water contained in the vessel is more than 10 litres. We perform this experiment by emptying the vessel by means of a siphon and collecting the water in a reference vessel of 10 litres. We give the answer \{yes\} when the water that is flowing from the first vessel to the reference vessel depasses 10 litres, and we give the answer \{no\} if this is not the case.

**experiment \( \beta \):** it consists in testing whether the depth of the water in the vessel is more than 15 cm. To perform this experiment we use a tube with a movable piece of wood in the tube. If we put the wood vertically in the vessel till we reach the bottom, the piece of wood will float on the water. When we redraw the tube out of the water, the depth of the water will be indicated by the position of the piece of wood in the tube. We give an answer \{yes\} if we read a depth of more than 15 cm. We give an answer \{no\} if this is not the case.