Fractal Dimensions and $1/f$ Noise.

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It is well known (1,2) that the noise spectrum in metal films and in carbon resistors shows a power law behaviour as a function of frequency with an exponent near to $-1$. This is at variance with the Lorentzian form expected on the basis of simple arguments of statistical mechanics. Many phenomenological theories have been put forward in order to explain this phenomenon within equilibrium statistical mechanics. In some such theories (3) a linear superposition of Gaussian noises is postulated, with a suitable relaxation time distribution. In our opinion it is quite difficult to understand the physical origin of such a distribution.

Some attempts to explain $1/f$ noise are based on dimensional or similarity arguments (4) following the philosophy of Kolmogorov's theory of fully developed turbulence (5). Other feature perturbative techniques—developed for turbulence—applied to the investigation of the motion of electrons in the resistor (viewed as a charged fluid) (6). These approaches still show some difficulties, due among other things to the hypothesis that the $1/f$ noise be a nonequilibrium phenomenon (which cannot yet be ruled out, but appears unconvincing in view of recent experimental findings (7)); they present, moreover, the shortcoming (common also to theories based on equilibrium mechanisms) of not explaining the rôle of the system dimensionality.

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(5) A. N. Kolmogorov: *Dokl. Akad. Nauk SSSR*, 30, 301 (1941); 31, 538 (1941); 32, 16 (1941).
The theoretical mechanisms so far invoked do not predict 1/f noise for some particular dimensionality, but in any dimension, which is at variance with the absence of 1/f noise in the usual, three-dimensional, resistors.

Let us consider the current circulating in the resistor as a charged, weakly compressible fluid. One should apply to it the magnetohydrodynamics equations (7, 9). It is convenient to our purposes to introduce the Fourier transform of the current density field:

\[
J(r) = \sum_k J_k \exp(-i k \cdot r).
\]

Following a general practice in the theory of fully developed turbulence (10), we introduce a "reduced variable" \( J_n \) which represents the fluctuations whose wave vectors \( k \) belong to the spherical shell \( A_n : k_n < |k| < k_{n+1} \), where \( k_n = a^n k_0 \) (\( a > 1 \), \( n = 0, 1, \ldots \)), and \( k_0 \) is the inverse of the linear extension of the sample:

\[
J_n = \int \frac{dk}{k} J_k^{(i)} ,
\]

where \( J_k^{(i)} \) is one of the components of \( J_k \). We remark that the introduction of such variables is just a help to the calculation and does not imply an analogy between 1/f noise and turbulence. The equations of motion for the reduced variables become

\[
J_n = F_n[J] - \nu J_n + \gamma \varepsilon_n ,
\]

where \( F_n[J] \) comes from the transport term in the original equation and contains nonlinear couplings in the form \( k_n J_n J_{n+1} \). The "reduced variable" \( \varepsilon \) for the electric field is indicated by \( \varepsilon_n \); \( \nu \) and \( \gamma \) are constants. For frequencies which are much lower than the plasma frequency the transport term \( F_n[J] \) is negligible. \( \varepsilon_n \) is related to the charge density fluctuations by the Poisson equation:

\[
\text{div} \varepsilon = \frac{1}{\varepsilon} \varrho .
\]

We obtain, therefore,

\[
\varepsilon_n \propto \frac{\varrho_n}{k_n} .
\]

In order to estimate the charge density fluctuations \( \varrho_n \), let us suppose that they are due to a large number of small scale-independent fluctuations. Such small-scale fluctuations have been recently observed (11). The central-limit theorem yields

\[
\left< \varepsilon_n^2 \right> \propto v_n^{-1} ,
\]