Summary. — It is shown that under certain assumptions unitarity demands the presence of parity twins for fermion trajectories, even if there is a fixed \( J \)-plane branch cut (Carlitz-Kislinger cut). The parity twin trajectory will not be degenerate with the original one unless the cut is absent, but for strong coupling unitarity makes it approximately degenerate. A modified Carlitz-Kislinger model is written down, which respects unitarity by including the parity twin; in the limit where the twin is degenerate the \( J \)-plane cut disappears.

1. -- Introduction.

It was observed a long time ago by Gribov \(^{(1)}\) that any fermion Regge pole must, at least if its trajectory does not intersect a \( J \)-plane branch point, be accompanied by a second Regge pole of opposite parity. The trajectory and residue of this "parity twin" must in fact be the analytic continuations of those of the original pole so that we have, say for the \( \Delta \) trajectory and its twin,

\[
\begin{align*}
\chi_{\text{tw}}(-\sqrt{u}) &= \chi_{\Delta}(\sqrt{u}) , \\
\beta_{\text{tw}}(-\sqrt{u}) &= \beta_{\Delta}(\sqrt{u}) .
\end{align*}
\]

\(^{(1)}\) To speed up publication, the author of this paper has agreed to not receive the proofs for correction.

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\(^{(1)}\) V. N. Gribov: Sov. Phys. JETP, 16, 1080 (1963); V. Singh: Phys. Rev., 129, 1889 (1963). The result is an immediate consequence of the MacDowell analyticity \(^{(2)}\) of meson-baryon partial-wave amplitudes, provided that there is no \( J \)-plane cut through which the \( J \)-plane Regge pole can disappear as \( \sqrt{u} \) is decreased through zero.

This result is embarrassing for the $\Delta$ trajectory because it appears experimentally (3) that for $\sqrt{u} \geq 0$, $\alpha_{\Delta}(\sqrt{u})$ is approximately a straight line in $u$

$$
\alpha_{\Delta}(\sqrt{u}) \simeq \alpha_0 + (\sqrt{u})^2,
$$

whereas if this form is retained for $\sqrt{u} < 0$ one would expect parity partners for the $\Delta$ and $F_{37}$ (which are not seen experimentally). Therefore one must either have a highly asymmetric trajectory (4) (Fig. 1 a)) or a residue function with zeros (5) whenever $\sqrt{u} = -\sqrt{\text{integer} + \frac{1}{2}}$

![Diagram](image)

Fig. 1. $\Delta$ and twin trajectories. a) No $J$-plane cut: i) $\Delta$ twin trajectory, and ii) $J$-plane singularities. This illustrates the usual picture, where the twin trajectory $\alpha_{tw}$ is identical with the $\Delta$ trajectory $\alpha_{\Delta}$. The asymmetric trajectory shown is necessary to eliminate the $\Delta$ parity partner, unless the residue $\beta_{\Delta}$ has an appropriate zero. b) Unitarity-violating Carlitz-Kislinger cut. The proposal here is that the unwanted part of the $\Delta$ trajectory is placed on a second sheet of the $J$-plane (dashed line). c) Unitarity-respecting Carlitz-Kislinger cut. The twin trajectory is necessary to avoid violating the unitarity bound. Its presence will reduce the discontinuity across the $J$-plane cut, but will not eliminate this cut unless it is identical with the $\Delta$ trajectory as in a). It must touch the $\Delta$ trajectory at $u = 0$, but has been separated there for clarity.

(3) Three points on the trajectory are established; the physical points corresponding to $F_{37}$ and $F_{37}$, plus the point at $u \approx 0$ obtainable from the energy dependence of backward $\pi^-p$ scattering (this latter is not affected by the Carlitz-Kislinger cut to be discussed, because the cut does not affect the energy dependence in the extreme backward direction). These three points are consistent with eq. (3) in units of (GeV)$^2$, with $\alpha_0 \approx 0.15$.


(5) This «asymmetric residue» mechanism is the usual one; for a recent study see: P. Minkowski: Linear fermion trajectories and absence of McDowell doublets, Zürich preprint (1970).