A Systematic Study of Strongly Localized Almost Massless Systems.

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Summary. — Properties of strongly localized almost massless clusters of constituents are systematically studied in the potential model of independent particles. The potential has a vector and a scalar part. For appropriate choices of potential, constituents are ultrarelativistic, strongly localized and almost massless \((r)E<10^{-6}\) with the anomalous magnetic moment proportional to \(e(r)\) —properties characteristic of quarks and leptons in constituent models. It is shown that rather large choices of potentials with vector and scalar parts enable such properties.

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1. – Introduction.

Several indications support the idea that quarks and leptons are clusters of constituents[1-3], although there is no direct experimental evidence for such models. Quarks and leptons have in constituent models very peculiar properties[4] in comparison with other known clusters. In particular, the product of the energy \((*)\) and the size of quarks or leptons is smaller than \(10^{-6}\), while for all other known systems this product is of the order 1 or greater.

It was shown in ref. [4] that the experimental spectrum of charged leptons and their electromagnetic properties can be reproduced in the independent particle model if a hypercolour interaction is postulated which manifests in the «long» distance sector as an effective potential with appropriate scalar and vector parts.

On the other hand the analogy with the (almost) massless pion, the Goldstone boson, which has even in the chiral limit the finite size \([5, 6]\) and therefore the product of the energy and the size infinitesimally small at least in the (nonrenormalizable) Nambu and Jona Lasinio model (in the limit when the cut-off \(\lambda\) goes to large values),

(*) We use the units for which \(h = c = 1\).
supports the idea, suggested in ref. [7], that quarks and leptons could be interpreted as quasi-Goldstone fermions. In this case an (almost) infinite medium of massless particles—a vacuum—is essentially needed. The correlated motion of particles in a vacuum causes coherent states of massive particles. Collective states, which are (almost) massless, are generated on such a vacuum. Several authors [7-9] have tried to reproduce the spectrum of the three families of quarks and leptons by choosing an appropriate (not necessarily supersymmetric) group determining the symmetries of the Lagrangian and a smaller subgroup, which determines the symmetries of a vacuum state. In such models fermions usually obtain their masses through Yukawa couplings. These models do not yet seem realistic. In our opinion, a systematic study of dynamics (in the sense it has been performed to some extent in the pion case [10]) may help to simplify the problem, since the single-particle degrees of freedom may contribute essentially to the formation of spectra of quarks and leptons (in addition to collective degrees of freedom—Goldstone fermions).

There are, however, several unsolved problems connected with bound systems of many relativistic fermions. Some of these problems were discussed in ref. [4].

This paper is in some sense the continuation of paper [4]. We present here a systematic study of the dependence of the ground-state properties of the clusters of constituents upon the choice of effective potentials. We show that a combination of a very strong vector potential of type $a'/r$ and a strong confining scalar potential—an infinite wall at small enough radius—guarantees a small enough product of the energy $E$ and the size $\langle r \rangle$ of the clusters ($\langle r \rangle E < 10^{-6}$) and the small enough contribution of the internal motion to the centre-of-mass magnetic moment. We show also that any strong attractive vector potential, with a finite value at the origin together with any scalar potential, confining the system to small enough size, can also do the same.

As in ref. [4], the independent particle model is used, so that the Dirac particle in an effective potential is studied. We assume that fermions carry, in addition to spin, the electric and the colour charge, a hypercolour charge which manifests in the long-distance region ($r \geq 10^{-4}$fm) as an effective potential with a vector and a scalar part in a similar sense as the colour interaction manifests itself on the hadron level as a scalar potential. Hypergluonic and hypermesonic fields are supposed to be much stronger than are the gluonic and the mesonic fields on the hadron level so that the confinement to a size smaller than $10^{-3}$fm is guaranteed. We do not discuss the origin of the effective potential (the complete derivation of which is not yet done even on the hadron level).

The paper is arranged as follows: in sect. 2 the equations of motion and the expressions for quantities like the energy, the magnetic moment, the mean radius, the Fourier components of wave functions are presented. In sect. 3 these quantities are evaluated and discussed as functions of strengths of vector and scalar potentials of a Coulomb type and as a function of the position of an infinite scalar wall or a step function. In subsect. 3'3 the properties of systems with finite values of the vector and the scalar potential at $r = 0$ are studied. In sect. 4 the conclusions are presented.

2. – The equation of motion.

The bound state of many fermion systems interacting through fields can in principle be solved using the Bethe-Salpeter equations. This is, however, a very difficult