On Differential Properties of Feynman Integrals.

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Summary. — It is shown how the differential properties of one-loop generalized (with Speer λ-parameters) Feynman integrals free from second-type singularities lead to the complete characterization of the analytic properties of these functions.

1. — Introduction.

The present paper completes a previous work (1) on the investigation of analytic properties of Feynman integrals with Speer λ-variables (2) relative to one-loop graphs having N external lines. In (1) we restricted our attention to integrals affected by Landau and second-type singularities (3); we gave the differential system satisfied by the above-mentioned integrals and pointed out how Lappo-Danilevsky's (4) theory works to construct the monodromy group of the solutions of the differential system.

Here we consider the same problem in the apparently simpler case of one-loop integrals free from second-type singularities. It will become clear that the procedure which one has to follow in this case only resembles the one used in (1).

Having set down the notations and basic properties in Sect. 2, we treat in Sect. 3 a particular class of functions \( (\lambda_{x+1} = 0) \), finding the corresponding differential system. In Sect. 4 the general case is shown to be reducible to the particular one studied in Sect. 3. Finally the Appendix contains explicit results which apply to the above-mentioned particular functions.

2. – Basic notations and properties.

For the sake of completeness we shall specify the basic notations and definitions; they will differ very little from those used in (1).

The integral we are interested in is

\[
I(A, \lambda) = \int_{\gamma_0} \prod_{\gamma \in \mathcal{P}(x+1)} x_i^{\lambda_i} \left[ B(x, A) \right]^{-\mu},
\]

where

\[
\mathcal{P} = \{1, 2, \ldots, N\},
\]

\[
B(x, A) = \sum_{i,j=1}^N x_i x_j A_{ij}, \quad A_{ij} = A_{ji}, \quad \forall i, j \in \mathcal{P},
\]

\[
s^{-1} \eta = \sum_{\gamma \in \mathcal{P}} (-1)^i x_i dx_1 dx_2 \ldots dx_{i-1} dx_{i+1} \ldots dx_N,
\]

\[
x_{n+1} = - \sum_{i \in \mathcal{P}} x_i, \quad 2\mu = N + \sum_{\gamma \in \mathcal{P}(x+1)} \lambda_i
\]

and

(2.2) \[
\lambda_{x+1} = M > 0, \text{ integer}.
\]

The integration is performed over a cycle \( \gamma_0 \) in \( \alpha \)-space, over which the integrand is regular. The Speer \( \lambda \)-variables and \( A_{ij}, \forall i, j \in \mathcal{P} \), are complex and, supposedly, independent. If we go over from the \( A \)-variables to the \( s_i \) Cayley variables (eq. (2.4) of (1)) it is easily checked that \( I(A, \lambda) \) takes the same form of the function \( F(s, \lambda) \) studied in (1), with only one essential difference, namely condition (2.2), which rules out the so-called 2nd-type singularities.