Schwinger Terms in Perturbation Theory (*).

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Summary. — Schwinger terms in current commutators are determined in a quark model, using perturbation theory. It is found that the equal-time commutator of two time components of vector currents contains no Schwinger terms, whereas the commutator of a time and a space component of vector currents contains a Schwinger term proportional to the derivative of a δ-function. The commutators of time components of axial and vector currents are also free from Schwinger terms, so that the local $SU_3 \times SU_3$ algebra recently proposed by Gell-Mann is unaffected.

In a previous paper (1) (hereafter referred to as I) it was shown that perturbation expansions for local currents in the quark model of Johnson and Low (2) satisfied the conventional commutation relations of current algebras, as derived from a free quark model, up to possible Schwinger terms which are infinite c-numbers, if a certain cut-off procedure, which seems natural, was adopted.

In this note we shall investigate the exact form of these Schwinger terms. We will be able to derive Schwinger terms for a general equal-time commutator of two local currents. The result agrees with the one of Schwinger (3) and Okubo (4) and it shows in addition that the local $SU_3 \times SU_3$ algebra of Dashen

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and Gell-Mann (5) (i.e. the algebra generated by the time components of vector and axial vector currents) is free from Schwinger terms.

In order to give a perturbation theoretic treatment of Schwinger terms in current commutators it is necessary to define very carefully what we mean by a current. It is customary to derive commutation relations for expressions like \( \psi(x) \gamma_\mu A \psi(x) \) from a free quark model and postulate these to hold for physical currents of the corresponding quantum numbers. But \( \psi(x) \gamma_\mu A \psi(x) \) is a bad candidate for our purposes since it is ill defined. In quantum electrodynamics the electromagnetic current is in fact defined as a normal product

\[
(1) \quad j_\alpha(x) = N(\psi^\dagger(x) \gamma_\mu \gamma_\alpha \gamma_\mu \psi(x)).
\]

We shall generalize this expression and represent currents in perturbation theory by

\[
(2) \quad j_i(x) = N(\psi^\dagger(x) \gamma_\mu A \psi(x)),
\]

where \( A_i \) is a direct product of \( \gamma \)-matrices and \( SU_3 \)-matrices. This expression has the advantage of being well-defined. There is also hardly any other choice but (2) since (2) contains the electromagnetic current as well, for which, we know, (1) must hold in analogy to quantum electrodynamics.

This precaution was irrelevant in I since the only difference between (2) and the expression \( \psi^\dagger(x) \gamma_\mu A \psi(x) \) formally reduces to an infinite additive constant, showing up in vacuum expectation values only; and these were not treated in I.

A further advantage of feeding (2) into a perturbation expansion lies in the fact, that diagrams like Fig. 1 are excluded; there are no contractions of two factors which are already in normally ordered form.

\[
\text{Fig. 1.}
\]

Our aim is to derive the commutation relations of the currents (2) from perturbation theory. Since the only difference between (2) and the currents used in I lies in the vacuum expectation values, we know already that the currents of (2) satisfy the conventional commutation relations up to possible Schwinger terms. These additional terms contributing to the vacuum expec-