Compositeness Criterion on an Unphysical Sheet (*).

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Summary. — The renormalization of a quantized field for which there exists no corresponding particlelike excitation is discussed in general terms. Since the absence of a single-particle term in the Lehmann representation of the propagator is associated with the passage of the pole which represents the discrete-state contribution through the elastic threshold, one can continue to carry out mass and wave function renormalization in the usual way by following the motion of the relevant pole to an unphysical sheet. It is shown that this leads to a definition of $Z_3$ for virtual states which allows it to be negative and unbounded, a circumstance which is in sharp contrast to the corresponding result for physical particles. The $Z = 0$ limit is discussed, it being demonstrated that an «elementary virtual particle» becomes a «composite virtual particle» in that limit.

1. – Introduction.

As far back as the days of the earliest successes of quantum electrodynamics, it has been more or less customary to make relatively little distinction between the dual concepts of particle and field. Thus one has generally tended to associate with each stable particle and each resonance (provided that the latter can be described by a measurable lifetime) a field of the appropriate spin and internal quantum numbers. Although a considerable amount of success has been attained in accomodating atoms and nuclei within the framework of a bound-state approach (thereby eliminating the necessity for an enormous increase in the number of elementary fields) the possibility of constructing such a bound-state picture has yet to be demonstrated in the realm of high-energy

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physics. While there has certainly been no lack of imagination displayed in
the many approaches to the problem of formulating a theory in which the vast
majority of particles can be considered bound states of some minimal set, the
proposals which have been advanced to date remain entirely speculative, of-
fering little or no basis for choosing between them.

One possibly unfortunate aspect of this tendency to identify particle and
field consists in the fact that it generally discourages the consideration of
fields which do not have a direct particle manifestation. We have in mind
here the situation in which the dynamics of a given theory does not allow the
association of either a resonance or a stable particle with a given field. Although
these are generally the circumstances under which such a field would not be
expected to have an appreciable effect upon low-energy processes, there does
exist a situation to which this conclusion does not necessarily apply. In par-
ticular it is possible that a virtual particle could occur in the vicinity of the
elastic threshold which might have a significant effect upon low-energy processes.
It therefore appears appropriate to discuss here some of the problems associated
with «particles» of this type which could conceivably be detected when one
has available more extensive scattering data than exist at the present time.

One of the most obvious questions raised by this possibility is the matter
of how to carry out the usual perturbation-theory calculations. The crucial
point here is the renormalization problem since in the absence of a pole on
the physical sheet there is no obvious subtraction point and one must conse-
quently make recourse to one of the unphysical sheets of the associated
Riemann surface.

The following Section briefly reviews the usual procedures essential to the
analytic continuation of the virtual-particle propagation through the lowest-
lying two-particle cut. In Sect. 3 the renormalization is carried out and the
properties of the wave function renormalization constant discussed. Finally
in Sect. 4 the $\mathcal{Z} = 0$ limit is examined on the unphysical sheet, it being shown
that the so-called $\mathcal{Z} = 0$ rule is valid in the case of a virtual particle as well
as in the more familiar application to stable particles.

2. – Analytic continuation.

Since our main interest here will be directed to the case in which there is
no stable particle associated with the field under consideration, at least at the
outset it will be necessary to work entirely with unrenormalized quantities.
We therefore define the propagator of the unrenormalized spin-zero field $\varphi(x)$
to be the time-ordered vacuum expectation value

$$G(x) = i\langle 0 | (\varphi(x)\varphi(0))_+ | 0 \rangle$$