Some Remarks on the Location of Singularities in S-Matrix Theory.

D. A. JACOBSON

Douglas Advanced Research Laboratories - Huntington Beach, Cal.

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Summary. — It is shown that the standard procedure for locating singularities in S-matrix theory through the analytic continuation of the unitarity relation (making use of the crossing relations and Hermitian analyticity), is not sufficient to determine upon which Riemann sheet certain «nonintersecting» singularities lie. The ambiguity is removed by assuming all such nonintersecting singularities absent from the physical sheet. Arguments are given which suggest that this assumption may be sufficient to «complete» the proofs of Hermitian analyticity and the crossing relations. Finally, it is shown that the assumption follows from a physical-sheet postulate which represents a strengthening of the usual iε-prescription.

1. — Introduction.

In analytic S-matrix theory, the singularities of scattering amplitudes are supposedly determined by an interplay between unitarity, Hermitian analyticity and the crossing relations. To be specific, using Hermitian analyticity, the amplitudes occurring in the integrand of a unitary integral can be written as boundary values or restrictions of analytic functions. This allows the unitary integral to be analytically continued. Using the crossing relations, and beginning with the normal threshold singularities which are implied by unitarity alone, the full set of Landau singularities \(^{(1)}\) are then generated by an iteration procedure. That is, assuming the amplitudes in the integrand of a unitary integral to have specific normal threshold singularities leads, through analytic

continuation, to new singularities of the integral, which may then be resubstituted into the integrand of a unitary integral to obtain still further singularities.

As explained more fully in the succeeding sections, the Landau singularities so generated are at first only found for the unitary integrals which are in turn related to the difference between an amplitude on two of its Riemann sheets. The problem remains therefore to determine the manner in which the singularity is distributed among its Riemann sheets, and in particular to see if this distribution agrees with that of perturbation theory. This has turned out to be a rather difficult problem, and especially difficult for anomalous thresholds (2). Recently however, OLIVE and LANDSHOFF (3) have shown that at least for real singularities, that is singularities for which all variables are real, the sheet distribution can be determined, apparently even for anomalous thresholds.

The purpose of this paper is to 1) show that this procedure cannot be extended to essentially complex singularities, thus leading to an apparent ambiguity in the above iteration procedure, 2) to suggest a possible procedure for removing this ambiguity, and 3) to indicate how this procedure is related to other properties of S-matrix theory.

2. – Source of the ambiguity.

Let \( A \) denote the connected part of a scattering amplitude involving a total of \( N \) particles. The crossing relations will be assumed to hold so that \( A \) represents several different but related processes. Let \( P \) denote the physical region for \( A \), a real environment of a connected \((3N-10)\)-dimensional complex analytic set \( C \) if scalar variables are used (4). The connected components of \( P \) consist of the physical regions for the various related processes which \( A \) describes. In order that the crossing relations have a nontrivial significance, it is of course necessary to be able to analytically continue \( A \) from one component of \( P \) to any other component of \( P \).

Denote the connected components of \( P \) by \( P_\alpha \), \( \alpha = 1, 2, ..., N_a \). For each component \( P_\alpha \), there is a unitarity relation for \( A \) of the form

\[
A(t) - A^*(t) = I^\alpha(t),
\]

where \( I^\alpha(t) \) is a sum of a finite number of unitary integrals, each of which contains a finite number of connected parts of scattering amplitudes. Equa-

