Baryon Spectroscopy and Regularities between $SU_6$ and $SU_3$
Mass-Breaking Parameters.

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When meson masses are studied with linear mass formulae, a link appears between the breaking of $SU_6$ and that of $SU_3$. For instance, $SU_6$ is strongly broken for the $L = 0$ mesons: $M_8(V) - M_8(P) \simeq 500$ MeV ($M_8$ is the octet mass when the $SU_3$-breaking term is switched off). Correlatively, we verify that the parameters which describe the breaking of $SU_3$ have very different values:

$$\delta m(V) \simeq 44 \text{ MeV}, \quad \delta m(P) \simeq 119 \text{ MeV} \quad (\delta m(P) = (M_K - M_\pi)/3).$$

In ref. (1) we used this correlation to make a choice among the various $I = Y = 0$ mesons, candidates to the $1^{++}$ nonet. In this letter we want to point out that this correlation exists also for the baryons, and we shall propose an explanation of this fact within the frame of the quark model (2).

We define our notations by recalling some well-known facts. The harmonic-oscillator symmetrical quark model (3) describes a state of three quarks by its total orbital momentum $L$ and by its $SU_6$ symmetry ($**$) $[M]$ ($M = 20, 56, 70$). Within a $L$-multiplet, the baryons are classified following the total spin $S$ of the quarks, the total angular momentum $J$ and the $SU_3$ symmetry $[N]$ ($N = 1, 8, 10$). We shall denote a given baryon by $B_{e+1}(N) f^P[M, L]$, where $P$ is its parity and $B$ its name. Let $V$ be the 2- and 3-body forces which break $SU_6$ leaving the $SU_3$ multiplets degenerate. They must contain spin-spin, spin-orbit, tensor forces, etc., in order to explain the experimental

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(1) M. Fontannaz: Mass difference within $SU_3$ multiplets and relativistic quark model, preprint LPTHE 72/30 Orsay.


(**) Another quantum number is also necessary for higher states which are not studied in this letter.
spectrum. We write (\*\) them

\[
V(1, 2, 3) = \sum_\alpha V_\alpha(|\rho|, |\eta|) C_\alpha(l_i, \sigma_i, \hat{Q}, \hat{\eta}, \lambda_i^{(n)}).
\]

\(l_i\) and \(\sigma_i\) (\(i = 1, 2, 3\)) are the orbital and spin momentum of quark \(i\); \(\rho\) and \(\eta\) are relative co-ordinates (\(\hat{Q} = \rho/|\rho|\)); the \(\lambda_i^{(n)}\) are the usual \(SU_3\) matrices (\(\dagger\)). Therefore, the perturbation of the masses in a \(L\)-multiplet is given (\(\dagger\)) by

\[
\delta M(ML, NSJ) = \langle ML, NSJ | V(1, 2, 3) | ML, NSJ \rangle = \sum_\alpha V_\alpha(ML) C_\alpha(ML, NSJ),
\]

where only the radial parts of the wave functions are used in the calculation of \(V_\alpha(ML)\).

Let us now study the \(SU_3\)-breaking mechanism. We assume that the breaking comes exclusively from the breaking of the quark mass (we put down \(m_\rho = m_\eta = m - \Delta m\)). But we also assume that the forces (\(\dagger\)) depend on the quark mass (\(\ddagger\)). Thus a modification of the latter induces a variation of the forces (\(\dagger\)), given to first order in \(\Delta m\) by

\[
\Delta V = -\Delta m \sqrt{3} \sum_{k=1}^{3} \lambda_k^{(8)} \frac{\partial V(1, 2, 3)}{\partial m_k} = -\Delta m \sqrt{3} \sum_{k=1}^{3} \lambda_k^{(8)} \sum_\alpha \frac{\partial V_\alpha(|\rho|, |\eta|)}{\partial m_k} \cdot C_\alpha(l_i, \sigma_i, \hat{Q}, \hat{\eta}, \lambda_i^{(n)}).
\]

Therefore, the perturbation of the mass of baryon \(B\) is (\(\dagger\))

\[
\Delta M_B = \langle B, ML, NSJ | \Delta V | B, ML, NSJ \rangle = -\Delta m \sqrt{3} \sum_{i=1}^{3} (\varphi_B, \lambda_i^{(8)} \varphi_B) \sum_\alpha \left\langle \frac{\partial V_\alpha}{\partial m_1} \right\rangle_{LM} C_\alpha(ML, NSJ).
\]

In writing this expression of \(\Delta M_B\), we have not included the «additivity term»

\(-\Delta m \sqrt{3} \sum_{i=1}^{3} \lambda_i^{(8)}\), nor the derivative of the \(SU_6\)-invariant forces. These two terms would add to expression (4) contributions independent of \(N, S, J\). But, as we are only interested in the \((N, S, J)\)-dependence of \(\Delta M_B\), we leave these terms aside.

(\(\dagger\)) We assume that the parts of \(V\) containing \(SU_3\) Casimir operators come from the exchange of meson nonets between the quarks, giving terms proportional to

\[
\lambda_i \cdot \lambda_j - \sum_{n=0}^{8} \lambda_i^{(n)} \lambda_j^{(n)} \quad \text{with} \quad \lambda^{(8)} = \sqrt{\frac{5}{2}} I \quad \text{and} \quad i \neq j = 1, 2, 3.
\]

Otherwise relation (3) below could be more complicated. We have written forces \(V(1, 2, 3)\) as local potentials, but they may be more general.


(\(\ddagger\)) In expressions (2) and (4), the sum \(\sum_\alpha\) is a «short-cut» notation. Indeed for the octets we have the wave function \(\chi^t = (1/\sqrt{2}) (\varphi_+ \varphi_+ + \varphi_+ \varphi_-)\), where \(\varphi_+, \varphi_-\) are the unitary-spin wave functions and \(\psi_+, \psi_-\) the spin-space wave functions, respectively symmetrical (\(a\)) or antisymmetrical (\(a\)) under the exchange of, say, quarks 2 and 3. Therefore \(\langle \chi^t | V_\alpha C_{\alpha} | \chi^t \rangle\) must be written

\[
\frac{1}{2} \left\langle \varphi_+ | u_a \varphi_+ (\varphi_+ + \varphi_-) | u_a \varphi_+ \right\rangle = \langle \varphi_+ u_a \varphi_+ | V_\alpha C_\alpha \rangle = u_a \varphi_+ ,
\]

\(u_a\) being the unitary-spin part of \(C_\alpha\). The spin-space wave functions also may be linear combinations of different terms and \(\langle \varphi_+ u_a \varphi_+ | V_\alpha C_\alpha \rangle\) must be understood as a sum.

(\(\ddagger\)) We have in mind a model where the potentials which bind the quarks are due to all possible exchanges of gluons. They depend on the quark masses through the quark propagator (\(\dagger\)).