Similar Single-Difference Model and Its Algorithm for Solving GPS Monitoring Deformation Directly at Single Epoch

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1 Introduction

Presently, GPS techniques are widely used to monitor the deformation of all kinds of buildings and constructions. For the high-precision GPS deformation monitoring, the carrier phases are usually used as the basic observational techniques. Detecting and repairing the cycle slips correctly and solving the ambiguities are the keys to obtain the high-precise positioning results. In this paper, a new similar single-difference mathematical model (SSDM) and its algorithm are advanced to solve the GPS monitoring information directly in single epoch.

2 Mathematical model

In order to obtain reliable deformation data and ensure the accuracy of deformation monitoring, the following three characteristics can be usually found in the high-precision GPS deformation monitoring networks: (1) The base points are built in the places where the geologic condition is steady and easily preserved. That is to say, the deformations of fiducial points compared with that of monitoring points, can be neglected. (2) In order to avoid the instrument error affecting the deformation monitoring accuracy, the monitoring distance (distance between fiducial point and monitoring point) commonly is less than 3 km. (3) In order to obtain reliable deformation data, the monitoring network should be linked with the adjacent
high level GPS points and use precise baseline resolution software to process baselines in the first observation. According to these characteristics, the SSDM will be introduced in detail.

The coordinates in WGS-84 system of fiducial point P1 and monitoring point P2 have been obtained through the first observation. During the course of surveying the monitoring network, the fiducial point P1 is immovable and the monitoring point P2 has displacement. The deformed position of P1 is P3, and the deformation value is d.

At epoch t, the carrier phase for the satellite i on the fiducial point P1 and the deformed monitoring point P3 can be expressed using the following two formulae:

\[
\begin{align*}
\tilde{\rho}_i &= \lambda \phi_i + \lambda N_i - c \delta t_i + c \delta t' + h_i \sin \theta_i + \\
\Delta \delta n_{\text{trop}} - \Delta \delta n_{\text{mult}} - \Delta \delta n_{\text{ion}}
\end{align*}
\]

\[
\tilde{\rho}_3 = \lambda \phi_3 + \lambda N_3 - c \delta t_3 + c \delta t' + h_3 \sin \theta_3 + \\
\Delta \delta n_{\text{trop}} - \Delta \delta n_{\text{mult}} - \Delta \delta n_{\text{ion}}
\]

where \(\lambda\) is the wavelength of carrier wave, \(\phi\) is the carrier phase observation, \(N\) is the initial integer cycles, \(\rho\) is the geometric distance between satellite and station, \(\delta\) is the distance varying ratio, \(h\) is the height of the antenna, \(\theta\) is the altitude angle, \(h \cdot \sin \theta\) is the correction that the distance between satellite and the antenna phase center is corrected to the distance between satellite and the station's center. After all of the corrections have been computed or some influence have been eliminated, only the initial integer cycles \(N\) is unknown in the two formulae.

In the spatial quadrangle of Fig. 1, composed of P1, P2, P3, and satellite i, the deformation d of the monitoring point P2 can be written as

\[
d = \tilde{\rho}_2 - \tilde{\rho}_3 - b
\]

where b is the known baseline between P1 and P2, which has been obtained through the first observation, \(\rho\) is the calculated carrier phase observation from Eqs. (1) and (2). In order to obtain the deformation of point P2, project Eq. (3) to the three coordinate axis directions X, Y, Z. The following will take the deformation dX of P2 in X axis direction as an example to derive the mathematical model for solving the monitoring point deformation directly in single epoch.

Mapping Eq. (3) to X axis direction, then

\[
dX = \rho_{X_1} - \rho_{X_3} - X_{X_1, X_3}
\]

(4)

Substituting Eqs. (1) and (2) into Eq. (4), and considering the direction cosine i, we have

\[
dX = \lambda P_{\text{t}} \rho_{\text{p}_1} - \lambda P_{\text{t}} \rho_{\text{p}_3} - X_{X_1, X_3} = \\
\lambda (\lambda P_{\text{t}} N_{\text{p}_1} - \lambda P_{\text{t}} N_{\text{p}_3}) + \lambda P_{\text{t}} (\lambda P_{\text{t}} - c \delta t_i + c \delta t') + \lambda P_{\text{t}} (\lambda P_{\text{t}} - c \delta t_i + c \delta t') + (\lambda P_{\text{t}} h_2 \sin \theta_2 + (\lambda P_{\text{t}} h_2 \sin \theta_2 + (\lambda P_{\text{t}} h_2 \sin \theta_2 + (\lambda P_{\text{t}} h_2 \sin \theta_2 + (\lambda P_{\text{t}} h_2 \sin \theta_2 + (\lambda P_{\text{t}} h_2 \sin \theta_2 +
\]

\[
dX = \lambda P_{\text{t}} \rho_{\text{p}_1} - \lambda P_{\text{t}} \rho_{\text{p}_3} - X_{X_1, X_3} = \\
\lambda (\lambda P_{\text{t}} N_{\text{p}_1} - \lambda P_{\text{t}} N_{\text{p}_3}) + \lambda P_{\text{t}} (\lambda P_{\text{t}} - c \delta t_i + c \delta t') + \lambda P_{\text{t}} (\lambda P_{\text{t}} - c \delta t_i + c \delta t') + (\lambda P_{\text{t}} h_2 \sin \theta_2 + (\lambda P_{\text{t}} h_2 \sin \theta_2 + (\lambda P_{\text{t}} h_2 \sin \theta_2 + (\lambda P_{\text{t}} h_2 \sin \theta_2 + (\lambda P_{\text{t}} h_2 \sin \theta_2 + (\lambda P_{\text{t}} h_2 \sin \theta_2 +
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\[
dX = \lambda P_{\text{t}} \rho_{\text{p}_1} - \lambda P_{\text{t}} \rho_{\text{p}_3} - X_{X_1, X_3} = \\
\lambda (\lambda P_{\text{t}} N_{\text{p}_1} - \lambda P_{\text{t}} N_{\text{p}_3}) + \lambda P_{\text{t}} (\lambda P_{\text{t}} - c \delta t_i + c \delta t') + \lambda P_{\text{t}} (\lambda P_{\text{t}} - c \delta t_i + c \delta t') + (\lambda P_{\text{t}} h_2 \sin \theta_2 + (\lambda P_{\text{t}} h_2 \sin \theta_2 + (\lambda P_{\text{t}} h_2 \sin \theta_2 + (\lambda P_{\text{t}} h_2 \sin \theta_2 + (\lambda P_{\text{t}} h_2 \sin \theta_2 + (\lambda P_{\text{t}} h_2 \sin \theta_2 +
\]

\[
\begin{align*}
\Delta \delta n_{\text{trop}} - \Delta \delta n_{\text{mult}} - \Delta \delta n_{\text{ion}}
\end{align*}
\]

(5)

In Eq. (5), besides the unknown deformation, there are the initial integer cycles of carrier phase, so Eq. (5) can not be used to calculate the deformation directly. The first term on the right side of Eq. (5) can be written as

\[
\begin{align*}
\Delta \delta n_{\text{trop}} - \Delta \delta n_{\text{mult}} - \Delta \delta n_{\text{ion}}
\end{align*}
\]

(6)