A Condition for Vanishing Electromagnetic Self-Stress in Nonlinear Classical Electrodynamics (*)

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Summary. — A condition for a vanishing electromagnetic self-stress is established, which also leads to a finite electromagnetic self-energy. Maxwell's theory does not satisfy this requirement. A class of nonlinear theories, which includes that of Born and Infeld, satisfies the condition.

1. Introduction.

The attempts of Abraham and Lorentz (1-3) to consider the interaction of an electron with its own electromagnetic field led to two major difficulties. The electromagnetic momentum and energy did not transform as a four-vector under Lorentz transformations, and the electromagnetic self-energy became infinite in the limit of a point charge. The first difficulty may be traced to a nonvanishing electromagnetic self-stress, which inevitably accompanies theories of the electron based on Maxwell's equations. Rohrlich (3-4) has shown that the electromagnetic energy and momentum can always be constructed in a Lorentz-covariant manner, even if the electromagnetic self-stress does not vanish. Thus, a vanishing electromagnetic self-stress is a sufficient but not a

(*) This work has been supported by the Deutsche Forschungsgemeinschaft and by the Bundesministerium für Bildung und Wissenschaft.

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(3) F. Rohrlich: Classical Charged Particles (Reading, Mass., 1965).

necessary condition for covariance of the electromagnetic energy and momentum. Attributing finite dimensions to the electron allows the electromagnetic self-energy to remain finite but leads to a very complicated equation of motion for the electron (7). Moreover, the self-stresses give rise to forces which tend to expand the particle.

Poincaré (5) attempted to account for the stability of electrons by introducing stresses of nonelectromagnetic origin. Dirac's relativistic generalization (6) of the classical theory of the electron greatly simplified the equation of motion and circumvented the problem of an infinite electrostatic self-energy by subtracting unwanted terms. In Dirac's work the finite size of the electron appeared in a new sense. Signals could be transmitted faster than light through the interior of an electron.

Born and Infeld (7) suggested another approach. They considered the mass of an electron to be entirely of electromagnetic origin and formulated a nonlinear version of electrodynamics with the expressed intention of making the electromagnetic self-energy finite. The electromagnetic self-stress vanishes in their theory.

Using the Lagrangian formalism we establish a condition for a vanishing electromagnetic self-stress. This condition also leads to a finite electromagnetic self-energy. In the case of a point charge, Maxwell's theory does not satisfy the condition. A class of nonlinear theories, which includes that of Born and Infeld, satisfies the condition.

Lastly we should like to point out that the predictions of nonlinear electrodynamics will soon be subject to experimental tests. We have shown (8) that Born and Infeld's electrodynamics leads to observable differences in the energy eigenvalues of electrons bound to superheavy nuclei. Even if superheavy nuclei do not exist, observable effects should result in the collisions of very heavy ions.

2. - The stress–energy–momentum tensor.

We consider the mass of the electron to be of an entirely electromagnetic origin, which is equivalent to the statement that no source terms are present in the Lagrangian. We also insist upon gauge invariance; thus, the Lagrangian depends only on the electromagnetic fields $f_{\mu\nu}$, which are related to the elec-

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