An Elementary Derivation of Wilson's and Polyakov's Confinement Tests from the Hamiltonian Formulation.

G. Marchesini

Istituto di Fisica dell'Università - Parma, Italia
Istituto Nazionale di Fisica Nucleare - Sezione di Milano, Italia

E. Onofri

CERN - Geneva, Switzerland

(ricevuto il 22 Aprile 1981)

Summary. — Starting from the Hamiltonian approach to lattice gauge theory, we show how the correlation functions introduced by Wilson and Polyakov in the Lagrangian approach are related to the evolution operator projected in the sector of the Hilbert space corresponding to external quark-antiquark charges (meson sector), thus asymptotically related to the meson interquark potential. We generalize this to introduce correlation functions both in the Wilson and the Polyakov scheme which are related to the interquark potential for three external quarks (baryon potential).

1. — Introduction.

In the Euclidean formulation of gauge theories the main tools to study quark confinement are the Wilson loop (1), which has been extensively analysed by Monte Carlo simulations, and the correlation function introduced by Polyakov (2) in a finite-temperature scheme. In fact, as discussed in the literature (1-4), the asymptotic behaviour of these two Green's functions is related to the binding energy of a static quark-antiquark pair.

Although such a relation with the interquark potential is well accepted, we think it would be useful (at least for the nonspecialist) to have an elementary derivation of it based on the Hamiltonian formulation of lattice gauge theory.

Besides being pedagogically useful, our derivation can be easily generalized. In fact, we will define Green's functions whose behaviour is related to potentials between different charges, e.g. the potential between three quarks (baryon potential).

In order to establish the relation between the Wilson and Polyakov correlation functions and the interquark potential, the most natural way is to start from the Hamiltonian formulation of lattice gauge theory developed by Kogut and Susskind (5), in which external charges are introduced through Gauss' law. The full Hilbert space is then divided into sectors according to external-charge configurations and the interquark potential is the ground state in the appropriate sector. We show then that the two correlation functions of Wilson and Polyakov are related, through the usual transfer matrix method, to the evolution operator projected in the quark-antiquark sector. Thus for large (Euclidean) time evolution the interquark potential (ground state) is dominating.

In the case of Polyakov correlation, this connection is very direct: by performing the usual Feynman-Kac-Nelson procedure to relate the Hamiltonian and the Lagrangian formulation, we show that this Green's function is nothing else than the trace of the evolution operator projected in the quark-antiquark sector.

In the case of Wilson loop this connection is less direct. As discussed in ref. (4), this Green's function corresponds, in the Hamiltonian formulation, to the vacuum correlation of the string operator. In this case the projection of the evolution operator into the quark-antiquark sector is accomplished by the string operator which connects the vacuum with this sector of states only.

The previous discussion can be straightforwardly generalized to other sectors of the Hilbert space by a suitable choice of the projection operators; in this way we obtain the Green's functions in the Lagrangian formulation which are related to various colourless charge configurations (qqq, qqq̅, etc.).

In sect 2 we briefly review the Hamiltonian formalism for pure lattice gauge theories. In particular, we characterize the various sectors of the Hilbert space through the Gauss law and we construct the corresponding projection operators. In sect. 3 and 4 we relate the Polyakov and Wilson correlations to the static quark-antiquark potential and we obtain the corresponding correlation functions related to the baryon potential.