A Derivation of Veltman's Equations in Classical Field Theory.

C. BOUCHIAT, G. FLAMAND and J. M. KAPLAN

Laboratoire de Physique Théorique et Hautes Energies - Orsay (*)

(trievuto il 5 Dicembre 1966)

Summary. — Veltman's equations are derived from a generalized principle of minimality and a further condition on the strong interactions, within the framework of classical Lagrangian field theory. The Poisson bracket algebra of the charge densities holds without the latter supplementary condition but « Schwinger terms » may appear in the brackets involving space components.

We shall present a general derivation in a classical Lagrangian formalism (1) of the equations recently proposed by VELTMAN (2). These give the divergences of the weak vector and axial currents \( j^v_{\mu} \), \( j^A_{\mu} \) in the presence of electromagnetic and weak interactions. They read

\[
\begin{align*}
\partial v^v &= -gW^v \times j^v - gW^A \times j^v - eA \times j^v + D^v, \\
\partial j^A &= -gW^v \times j^A - gW^A \times j^A - eA \times j^A + D^A.
\end{align*}
\]

The cross product \( W \times j \) (and similarly \( A \times j \)) is defined by

\[
(W \times j)_a = f_{abc} W^b j^c_{\mu},
\]

where \( f_{abc} \) is the usual totally antisymmetric real tensor of the algebra \( SU_3 \).

(*) Postal address: Laboratoire de Physique Théorique et Hautes Energies, Bât. 211, Faculté des Sciences, 91 Orsay.

(1) After the completion of this work we became aware of a paper of Pasupathy where similar but less complete considerations are forwarded. J. PASUPATHY: University of Rochester preprint U.R. 875-162.

are the divergences of the vector and axial currents when the weak and electromagnetic interactions are turned off.

The sixteen vector bosons $W^a$, $W^\alpha$ in general will not be independent fields. The violation of parity in the form of the $eV-A$ theory is obtained by setting $W^\alpha_\mu = W^\alpha_\mu$. Depending on the kind of $W$ theory chosen the eight $W^a$ will be linear combinations of the true $W$-field operators. For example in a theory of Cabibbo's type with only two charged bosons, we have

\[
\begin{align*}
W^a &= 0 \quad \text{for } a = 3, 6, 7, 8, \\
W_1 &= \cos \theta \frac{W^+ + W^-}{\sqrt{2}}, \quad W_2 = i \cos \theta \frac{W^+ - W^-}{\sqrt{2}}, \\
W_4 &= \sin \theta \frac{W^+ + W^-}{\sqrt{2}}, \quad W_5 = i \sin \theta \frac{W^+ - W^-}{\sqrt{2}}.
\end{align*}
\]

The $A^{\mu a}$ are given in terms of the photon field by

\[
\begin{align*}
A^{\mu a} &= 0, \quad b = 1, 2, 4, 5, 6, 7, \\
A^{\mu a} &= \frac{1}{\sqrt{3}} A^a.
\end{align*}
\]

Let us call $\mathcal{L}(\psi_i, \partial_\mu \psi_i)$ the strong-interaction Lagrangian where $\psi_i$ stands for the hadron fields. We construct the Lagrangian $\mathcal{L}^{V-B}$ describing the strong, electromagnetic and weak interactions of the hadrons by the following generalized principle of minimality:

In the Lagrangian $\mathcal{L}(\psi_i, \partial_\mu \psi_i, \partial_\mu W, \partial_\mu W)$ is replaced by

\[
\chi_\mu = \partial_\mu \psi_i - g[F^a_\mu(\psi) W_\mu^a + F^{\alpha}_\mu(\psi) W_\mu^{\alpha}] - ie A_\mu.
\]

$F^a_\mu(\psi)$ and $F^{\alpha}_\mu(\psi)$ are functions of the hadron fields and not of their derivatives. We shall see that for the Veltman equations to hold true, the "gauge" functions $F(\psi)$ have to satisfy a set of differential equations which, as Poisson brackets, have the structure of the Lie algebra $SU_3 \otimes SU_3$.

In the following we shall forget about the electromagnetic interaction. This case has been considered previously by Adler and Coleman (3). The total Lagrangian $\mathcal{L}_i$ for the hadrons $W$-meson system is given by

\[
\mathcal{L}_i(\psi, \partial_\mu \psi, W, \partial_\mu W) = \mathcal{L}(\psi, \chi_\mu) + \mathcal{L}_o(W, \partial_\mu W),
\]

where $\mathcal{L}_o(W, \partial_\mu W)$ is the free Lagrangian of $W$-mesons.