«Handed» Particle of Spin $\frac{1}{2}$ with Finite Mass.

S. WATANABE

IBM Research Laboratory - Poughkeepsie, New York

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Summary. — It is possible to formulate a four-component spinor theory for a particle of spin $\frac{1}{2}$ with a definite chirality (handed-ness), which has a finite mass. Interaction between an achiral (nonhanded) spinor and a chiral spinor requires an eight-component framework. Such an interaction manifests a parity-violating property. Two formulations are possible: 1) The theory can be made invariant separately for time-reversal and for the combination of space-inversion and charge-conjugation, or 2) it can be made invariant for space-and-time-inversion and for charge-conjugation. The present theory may have applications for hyperons. The Salam-Lee-Yang-Landau type (1) of neutrino theory can be derived from the present formalism for the case $m = 0$. (See Appendix).

1. Chirality operator for particle with finite mass.

In the two-component neutrino theory (1), the useful two components can be extracted from the usual four-component spinor by the use of the chirality operator $\gamma_5$. In the same way, in the present theory, we shall first consider an eight-component spinor, from which we shall extract four components by the help of a chirality operator suitably defined.

According to the purely mathematical definition of a spinor, i.e., without the field-theoretical re-interpretation of time-reversal, the simple reflection of the $x_\mu$-co-ordinate ($\mu = 1, 2, 3, 4$) is represented by $\gamma_\mu \gamma_5$. The chirality operator $\gamma_5$ in the case of the neutrino theory is the simplest operator that anti-
commutes with each of $\gamma_\mu\gamma_5$ and commutes with the Hamiltonian. This fact will give a clue in defining the chirality operator in the present case.

We define $8 \times 8$-matrices $\Gamma_\mu$ ($\mu = 1, 2, 3, 4$) and $\Gamma_5$ exactly in the same way as the usual $\gamma$'s.

\[
[\Gamma_\mu, \Gamma_\nu]_\pm = 2\delta_{\mu\nu}, \quad \Gamma_5 = \Gamma_1\Gamma_2\Gamma_3\Gamma_4.
\]

The Hamiltonian is given by

\[
H = i\Gamma_4\Gamma_\alpha p_\alpha + \Gamma_4 m. \quad (a = 1, 2, 3).
\]

We cannot use $\Gamma_5$ as the chirality operator since it does not commute with $H$ unless $m = 0$. We shall presently see the way to overcome this difficulty.

In the system of $4 \times 4$-matrices, a matrix that commutes with all four $\gamma_\mu$ ($\mu = 1, 2, 3, 4$) is the unity matrix multiplied by an ordinary number. However, in the system of $8 \times 8$-matrices, a unitary Hermitian matrix $A$ exists which without being a multiple of the unity matrix commutes with all four $\Gamma_\mu$ ($\mu = 1, 2, 3, 4$).

\[
[A, \Gamma_\mu] = 0, \quad (\mu = 1, 2, 3, 4), \quad [A, A_\alpha] = 0,
\]

\[
A^2 = \overline{A}A = 1.
\]

According to the usual definition of spinors, the simple reflection with respect to a plane whose normal is $n_\mu$ ($n_\mu n_\mu = \pm 1$) is represented by the transformation matrix

\[
S = n_\mu \Gamma_\mu \Gamma_5.
\]

We can now change this to

\[
S = n_\mu \Gamma_\mu \Gamma_5 A,
\]

without in any way altering the transformation properties of the physical quantities on account of Eq. (1.3). Eq. (1.4) shows that the eigenvalues of $A$ are $\pm 1$. If we limit the wave-functions to those which are eigenfunctions of $A$ corresponding to one of the eigenvalues, then the transformation rule Eq. (1.6) resumes the familiar form of Eq. (1.5).

Passing to the general case, if $S_0$ is the transformation matrix according to the customary definition based on Eq. (1.5), then the one we use here can be written

\[
S = A^* S_0,
\]