Parametric Selection and Continual Combination of Building Surface

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Abstract: A method to reparametrize $G^1$ retional curve to obtain a $C^1$ curve is given. A practical $G^1$ continual connective between adjacent NURUS patches along common quadratic boundary curve is presented in this paper, and a specific algorithm for control points and weights of NURBS patches is discussed.

Key words: parametrization, cross-section design, surface modeling, non-uniform rational B-spline, geometric continuity

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0 Introduction

The method to design building surfaces by the cross-section contours enables designers to control the form of curves better. But if the cross-section line is not $C^1$ continual, the camber obtained will not meet the $G^1$ continuity, so we have to reparametrize the cross-section line.

The camber roof of high-story and superhigh-story buildings and then steel component of giant pot vessels are all welded together by lots of small cambers, and a single piece of camber can't meet the demand of contour design. So it's necessary to construct a combinative established camber, which has a certain continual degree, to design the continual integrative camber with any size and shape. The continual surface of the tangential plane ($G^1$) and the curvature continuity ($G^2$) are most frequently used in engineering. This paper offers a practical condition to continuously combine non-uniform rational B-spline (NURBS) surface plate $G^1$ which has the twicecommon-bordered curve, to obtain the relative controlling vertex and the concrete algorithm of the right coefficients. For a known $B$ strip surface, it's convinced and practical to construct another $B$ strip surface to make it meet $G^1$ glossy continuity.

1 Parametric Selection of the Cross-section Curve

As pointed out by Ref. [1], when defining a surface with a set of curves, parameterization of curves is very important. If the cross-section curve is not $C^1$ continual, then the skinning surface produced by the set of curves will not guarantee the $G^1$ continuity. So it's significant to reparametrize the cross-section curve for producing the continual skinning surface.

Defining the cross-section curve $C(t)$ of the $G^1$ continual surface is set as [2]:

\[
C(t) = \sum_{i=0}^{m} \sum_{j=0}^{n} w_{ij} C_{ij} B^m_j \frac{t-t_i}{t_{i+1}-t_i},
\]

\[
= \sum_{i=0}^{m} \sum_{j=0}^{n} a_{ij} B^m_j \frac{t-t_i}{t_{i+1}-t_i},
\]

$t \in [t_0, t_{m+1}]

Suppose $t_1 < t_2 < \cdots < t_n$ are $n$ non-$C^1$ continual points, We obtain $n+1$ pieces of subcurve sections $A_0(t), A_1(t), \cdots, A_n(t)$ by breaking up the curve $C(t)$ at $t_i$:

\[
A_i(t) = \sum_{j=0}^{m} a_{ij} C_{ij} B^m_j \frac{t-t_i}{t_{i+1}-t_i},
\]

\[
= \sum_{j=0}^{m} a_{ij} B^m_j \frac{t-t_i}{t_{i+1}-t_i},
\]

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For every sub-curve section \( A_i(t) \), seek a parameter to change to \( t = f_i(u) : [t_i, t_{i+1}] \rightarrow [t_i, t_{i+1}] \), then

\[
\frac{dA_{i-1}(t_{i-1})}{du(t_i)} = \frac{dA_i(t_i)}{du(t_i)}
\]

It will guarantee the combinative curve \( A_i(f_i(u)) \) is \( C^1 \) continual. As there are two kinds of cross-sections, broken-up curve and closed curve, so we discuss the parametric selection under two circumstances as below.

1.1 \( C(t) \) is a broken-up curve

When

\[
a_0 = 0, 1 + a = \frac{1}{1 + a_{i-1} \cdot \left\| \frac{dA_i(t_i)}{du(t_i)} \right\|}
\]

define parametric change as below

\[
f_i(u) = t_i + \frac{(1 + a_i)(u - t_i)}{a_i(u - t_i) + \Delta t_i} \cdot \Delta t_i,
\]

\[
\Delta t_i = t_{i+1} - t_i, i = 0, 1, \ldots, n
\]

Obviously, \( f_i(u) \) changes area \([t_i, t_{i+1}]\) into area \([t_i, t_{i+1}]\), make \( t = f_i(u)\), then the combinative curve \( C(u) = \{A_0(f_0(u)), \ldots, A_n(f_n(u))\} \) is \( C^1 \) continual in area \([t_0, t_{n+1}]\).

1.2 \( C(t) \) is a closed curve

When make \( a_i = \min \left\{ \frac{\left\| \frac{dA_i(t_i)}{du(t_i)} \right\|}{\left\| \frac{dA_i(t_{i+1})}{du(t_{i+1})} \right\|}, 1 \right\} \) define parametric change as below

\[
f_i(u) = t_i + A_i(t_{i+1}) \cdot \frac{u - t_i}{\Delta t_i}, i = 0, 1, \ldots, n
\]

as \( C(t) \) is a closed curve, so \( A_i(t) = A_n(t), A_{n+1}(t) = A_0(t) \). Suppose \( h_i(s) : [0, 1] \rightarrow [0, 1] \) under conditions \( h_i(0) = 0, h_i(1) = 1, h_i'(s) = a_i \) and the cubic polynomial of \( h_i'(1) = \beta_i \), then \( h_i(s) = (a_i + \beta_i - 2)s^2 + (3 - 2a_i - \beta_i)s + a_i \). So the parametric changing

\[
f_i(u) = s + \Delta t_i \cdot \frac{u - t_i}{\Delta t_i}, i = 0, 1, \ldots, n
\]

makes area \([t_i, t_{i+1}]\) into area \([t_i, t_{i+1}]\). With this parametric changing, the combinative curve \( C(u) = \{A_0(f_0(u)), \ldots, A_n(f_n(u))\} \) is \( C^1 \) continual in area \([t_0, t_{n+1}]\).

In order to make \( G^1 \) surface, we get \( C^1 \) continuity in area \([t_0, t_{n+1}]\) by using the above two methods to reparametrize the cross-section line of \( G^1 \) surface, we have to give proofs as below:

For a broken-up curve, as \( f'_i(u) = (\Delta t_i)^2(1 + a_i)(u - t_i) + \Delta t_i^2 \), obviously, \( f'_i(u) > 0 \).

For a closed curve \( f'_i(u) = h'_i(u - t_i)/\Delta t_i \), so we only need to testify that \( h'_i(s) > 0 \) in area \([0, 1]\).

After deducing \( h_i(s) \), we get \( h'_i(s) = 3(a_i + \beta_i - 2)s^2 + (3 - 2a_i - \beta_i)s + a_i \). Because \( 0 < a_i, \beta_i < 0 \), the quadratic term coefficient is negative. That's to say \( h'_i(s) \) has no minimum, and \( h'_i(0) = a_i > 0, h'_i(1) = \beta_i > 0 \), so \( h'_i(s) \) is definitely positive in area \([0, 1]\).

From above, \( f_i(u) \) increases monotonously in area \([t_i, t_{i+1}]\), that is \( f'_i(u) > 0 \), for any \( u \in \{t_i, t_{i+1}\} \).

2 \( G^1 \) Continual Combination of Building Surface

The parametric selection as above is suitable for the rational B surface constructed by the family of cross-section curves and for the Bezier surface as well, and the selection has a good function of regulation. In recent years, NURBS surface has been used in some advanced CAD systems, the major virtues are: it can accurately show and analyze free surfaces with a simple mathematical form; the defined right factors make the contour design more convenient and flexible; users can achieve the desired results by adjusting elements as points, lines, and surfaces which have object geometric sense. It includes non-rational B strip surface, rational and non-rational Bezier surface. So, this section discusses the algorithm to continue to combine two pieces of NURBS surface \( G^1 \).

The adequate and essential conditions of \( G^1 \) continuity which are met by two surfaces \( R(u, v), R(\bar{u}, \bar{v}) \), are the two surfaces having the common-bordered tangential plane continuity, that is\(^1\):\(^2\):

\[
\begin{align*}
R(u, \bar{v}) &= R(u, v) \quad (1) \\
R(\bar{u}, \bar{v}) &= aR_u(u, v) + \beta R_v(u, v) \quad (2)
\end{align*}
\]

Among above, \( a, \beta \) are arbitrary functions of \( v \), when two pieces of surfaces \( R(u, v), R(\bar{u}, \bar{v}) \) having common-border curves meet \( G^1 \) continuity, they must satisfy conditions as below:

\[
\begin{align*}
Q(0, \bar{v}) &= c_0(\bar{v})Q(0, v) \quad (3) \\
Q(\bar{u}, \bar{v}) &= c_1(\bar{v})Q(\bar{u}, v) + c_0(\bar{v})p_1(v)Q_0(\bar{u}, v) + c_1(\bar{v})q_1(v)Q_0(\bar{u}, v) \quad (4)
\end{align*}
\]

Among above, \( c_0(\bar{v}), c_1(\bar{v}), p_1(v), q_1(v) \) are functions of the common-border parameter \( v = \bar{v} \). \( Q(u, v), Q(\bar{u}, \bar{v}) \) are respectively even-timed coordi-